Error analysis and Modeling for High Accuracy Absolute Capacitive Displacement System with Long-Range

Dongdong Zhang 1,2,3, Li Lin 1,2,3,4,* and Quanshui Zheng 1,2,3,4

1 State Key Laboratory of Tribology, Tsinghua University, Beijing 100084, China;
2 Department of Mechanical Engineering, Tsinghua University, Beijing 100084, China;
3 Center for Nano and Micro Mechanics, Tsinghua University, Beijing 100084, China;
4 Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

* Correspondence: linli@mail.tsinghua.edu.cn

Received: date; Accepted: date; Published: date

Abstract: A novel kind of absolute capacitive grating displacement measuring system with both high-accuracy and long-range was proposed, which includes a MOVER and a STATOR. Both the contact surfaces of MOVER and STATOR are coated by a thin layer of dielectric film with low friction coefficient and high hardness. The measuring system works in a contact mode to minimize gap change. This paper presents theoretical analysis for the influence of some factors, including fabrication errors, installation errors and environment disturbance, on measurement signals. Measuring signal model is modified according to the analysis. Signal processing methods are investigated to improve signal sensitivity and signal-noise-ratio (SNR). Displacement calculation model shows that dead-zone problem is solved by orthogonal signals’ design. Absolute displacement is obtained by a simple method using two coarse signals and high accuracy displacement is further determined using two fine signals with the help of absolute information. According to displacement calculation model and error analysis, model accuracy is mainly determined by the error of fine calculation functions and locally affected by coarse calculation functions. It also can be drawn that amplitude difference, non-orthogonality and signal offset are not related to displacement accuracy. Experiments are carried out to confirm the theoretical analysis mentioned above. As demonstrative examples, experiment results show that the displacement resolution and accuracy respectively reach ±4.8 nm and ±34 nm in the displacement range of 5 mm. Both experiments and theoretical analyses indicate that measuring system have great potential of both tens of nanometers high-accuracy and hundreds of millimeters long-range.

Keywords: absolute displacement measurement; nanometer accuracy; long range; error analysis;

1. Introduction

Displacement measurement with nanoscale resolution and accuracy over a range of several hundred millimeters is crucial in many industrial fields such as semiconductor manufacturing and ultra-precision machining [1-4]. It is very challenge to achieve both high-accuracy and long-range measurement [5,6]. Among various kinds of displacement sensors [7-11], laser interferometers, grating rulers and capacitive grating sensors are the major types of transducers for displacement measurement with comparable high precision for relative long range [12-18]. Laser interferometers have a range of dozens of centimeters or even several meters with relative accuracy better than ±0.1 ppm [13,19-21]. In addition to the cost and complicated structure, they are sensitive to many factors such as beam interference, optical mixing, air temperature and humidity, variation in the optical
medium [22-24]. Due to the above drawbacks, they are only suitable for use in well-controlled environments like calibration applications [5,17,25,26]. Compared with laser interferometers, grating rulers are more insusceptible to environment, and they are universally used in workshop situations for high accuracy and long range requirement. The accuracy of commercial grating rulers is usually about ±1 μm, and the accuracy is quite difficult to be improved because of the restrictions of nanofabrication [27,28]. Due to advantages of simple structure, low cost, low power consumption and robustness to environment [8,29,30], capacitive grating displacement sensors are arranged with periodical electrodes trying to achieve both high precision and long range [31-33]. The measurement precision of such area-change based capacitive grating sensors is very vulnerable to gap change [16,26,34]. A contact-type sensor was proposed to reduce gap change error, but it wasn’t able to truly reach the goal of long range measurement due to the dead-zone regions existing in periodic signals where measurement is insensitive to displacement change resulting in major accuracy problem [35,36]. Another kind of time grating based sensor was reported that the resolution is not limited by the electrode pitch and gap. However, the accuracy of the sensors is affected by signal qualities including amplitude difference, phase difference and offset caused by error of fabrication and installation [17,25,37,38].

Absolute displacement sensors provide absolute position information immediately without searching references through motion in the condition of reboot after power loss [39,40]. Compared with incremental sensors, absolute displacement sensors remove accumulative errors and provide position information more efficient [41]. These characteristics of absolute displacement sensors are extremely essential especially for closed loop feedback control in industrial production. Generally, laser interferometers obtain absolute position using time-of-flight method or multi-wavelength method, but these methods are very complex [42-44]. Absolute grating rulers or grating encoders apply binary code patterns occupying one or more code tracks to detect absolute displacement information [45], but these code tracks are difficult to manufacture because of accuracy requirements [46]. It is also complex and time consuming to get absolute position information from grating rulers or grating encoders [47,48]. There are very few reports on absolute methods used for capacitive grating linear displacement sensors, and the absolute displacement accuracy is greatly influenced by the precision of periodic size of electrodes according to calculation principles [49,50].

An absolute capacitive grating displacement measuring system with both high-accuracy and long-range has been proposed, which includes a MOVER and a STATOR [16]. A thin layer of low friction coefficient dielectric film is coated on the contact surface of each the MOVER and the STATOR. The measuring system is working in a contact mode to minimize gap change error, and measurement accuracy is not affected by gap nonuniformity when MOVER moves relative to STATOR. A simple and novel absolute method is introduced to the measuring system, and this method ensures that measuring system accuracy will not be affected by accuracy of fabrication and installation. Dead-zone regions are firstly pointed out and two orthogonal periodic signals are selectively and alternately used to solve the problem.

In this paper, a signal model is modified based on basic measuring principle and theoretical analysis for the influence of fabrication errors, installation errors and environment disturbance on measurement signals. Signal processing methods are investigated to improve signal sensitivity and signal-noise-ratio (SNR). Displacement calculation model is established to get high accuracy absolute displacement and offer theoretical support for error analysis. Analysis of error and accuracy demonstrates the major sources of calculation model error and provides information for increasing model accuracy. According to displacement calculation model and error analysis, the measurement accuracy is independent of fabrication errors and installation errors. Experiments and analysis are carried out in the last section.

2. Basic Measuring Principle

Figure 1a demonstrates the overall structure of proposed displacement measuring system, which includes a MOVER and a STATOR [16]. Both the MOVER and the STATOR consist of periodically arranged electrodes covered with a thin layer of low friction coefficient dielectric film
The MOVER moves relative to the STATOR in a contact mode along the X direction. On the MOVER, there are two rows of metal electrodes labeled as Mc with only one electrode in each row, and another two rows of metal electrodes labeled as Mf with n (n=3) electrodes in each row. The width and length of electrodes Mc are $W_{fg}$ and $L_{fg}$, and interval between two adjacent electrodes is also $W_{fg}$. The width and length of electrodes Mf are $W_{cg}$ and $L_{cg}$. All these rectangular metal electrodes on MOVER are connected together. On the STATOR, four grating groups of electrodes labeled as A, B, C, D in the middle two rows are served as fine measurement, and the other four grating groups labeled as E, F, G, H in the sides two rows are employed as coarse measurement to provide absolute displacement information. As is shown in Figure 1b, the electrode width is $W_f$, the electrode length is $L_f$, and interval between two adjacent electrodes is $I_f$. The two rows of electrodes are offset by a distance of $W_{fg}/2$, and $W_{fg}$ is equal to $(W_f+I_f)$. The four grating groups A, B, C, D combing with electrodes Mc can respectively form variable capacitor groups (VCGs) labeled as $C_A$, $C_B$, $C_C$, $C_D$, and these four VCGs are used for fine measurement. As shown in Figure 1d, the electrode width is $W_c$, the electrode length is $L_c$, and interval between two adjacent electrodes is $I_c$. There is also a distance difference of $W_{cg}/2$ between the two rows of coarse electrodes, and $W_{cg}$ is equal to $(W_c+I_c)$. The four grating groups E, F, G, H combing with electrodes Mf can respectively form variable capacitor groups (VCGs) labeled as $C_E$, $C_F$, $C_G$, $C_H$, and these four VCGs are used for coarse measurement.

Figure 1. Schematic of displacement measuring system. (a) Overall structure of proposed displacement measuring system; (b) Top view of fine displacement measurement; (c) Section view of displacement measuring system; (d) Top view of coarse displacement measurement.
For an ideal parallel-plate capacitor, capacitance is determined by gap between two electrode plates, overlap area of two plates and dielectric properties of insulator between the plates. As shown in Figure 1, according to \( C = \varepsilon S/d \), \( C_A(x) \) is described as:

\[
C_A(x) = \begin{cases} 
\frac{n\varepsilon}{d}(W_f - x)L_f + C_{A}^0, & x \in [0, W_f) \\
C_{A}^0, & x \in [W_f, W_{fg}) \\
\frac{n\varepsilon}{d}(x - W_f - L_f)L_f + C_{A}^0, & x \in [W_{fg}, 2W_{fg} - L_f) \\
\frac{n\varepsilon}{d}W_fL_f + C_{A}^0, & x \in [2W_{fg} - L_f, 2W_{fg}]
\end{cases}
\]  

(1)

where, \( \varepsilon \) is permittivity of dielectric materials between metal electrodes on MOVER and STATOR, \( d \) is distance between metal electrodes on MOVER and STATOR and \( n \) is the electrode number of electrodes \( M \) in each row on MOVER. \( x \) is the relative displacement of MOVER and STATOR in the X direction. Due to the parasitic capacitance, constant \( C_A^0 \) is introduced in Equation (1).

To simplify Equation (1), it can be rewritten as:

\[
C_A(x) = C_f(x) + a
\]  

(2)

where

\[
C_f(x) = \begin{cases} 
\frac{2A_f}{W_f}x + A_f, & x \in [0, W_f) \\
-A_f, & x \in [W_f, W_{fg}) \\
\frac{2A_f}{W_f}x - A_f \left( 3 + \frac{2L_f}{W_f} \right), & x \in [W_{fg}, 2W_{fg} - L_f) \\
A_f & x \in [2W_{fg} - L_f, 2W_{fg}]
\end{cases}
\]

(3)

\[ a = A_f + C_{A}^0 \]  

(4)

In Equation (3) and Equation (4), \( A_f = \frac{n\varepsilon W_f L_f}{2d} \) is the amplitude of functions for fine measurement.

In fact, \( C_f(x) \) is a periodic function within the displacement range \( L \), whose period is \( T_f = 2W_{fg} \), corresponding to period of electrode arrangement for fine measurement.

For a more general form, Equation (2) can be further formulated as

\[
C_A(x) = C_f(x + \phi_f) + a
\]  

(5)

where, \( \phi_f \) is the initial phase of displacement, which means that displacement start at a certain location.

The function \( C_B(x) \) is derived in the same way. The location difference of grating group A and grating group B is \( W_{fg} \), thus:

\[
C_B(x) = C_f(x + W_{fg} + \phi_f) + b
\]

\[
= -C_f(x + \phi_f) + b
\]  

(6)

where, \( b = A_f + C_{A}^0 \). \( C_B^0 \) is parasitic capacitance, and its value may be a little different from \( C_A^0 \).

Similarly, function \( C_C(x) \) and \( C_D(x) \) are derived:

\[
\begin{align*}
C_C(x) &= +C_f(x - T_f/4 + \phi_f) + c \\
C_D(x) &= -C_f(x - T_f/4 + \phi_f) + d
\end{align*}
\]

(7)

where, \( c = A_f + C_{D}^0 \), \( d = A_f + C_{D}^0 \)

Structure design makes sure that functions \( C_E(x) - C_H(x) \) for coarse measurement have no more than a single cycle to ensure the uniqueness of measurement within the displacement measurement range \( L \). When \( C_E(x) - C_H(x) \) just have a cycle, whose period is \( T_c = L \), they can be expressed:

\[
\begin{align*}
C_E(x) &= +C_c(x + \phi_c) + e \\
C_F(x) &= -C_c(x + \phi_c) + f \\
C_G(x) &= +C_c(x - T_c/4 + \phi_c) + g \\
C_H(x) &= -C_c(x - T_c/4 + \phi_c) + h
\end{align*}
\]

(8)
where, \( e = A_c + C^0_c \), \( f = A_c + C^0_f \), \( g = A_c + C^0_g \), \( h = A_c + C^0_h \) and

\[
C_c(x) = \begin{cases} 
\frac{2A_c}{W_c} + A_c & \text{if } x > 0 \\
-A_c & \text{if } x < 0 \\
\frac{2A_c}{W_c} x - A_c \left(3 + \frac{2l_c}{W_c}\right) & \text{if } x \neq 0 
\end{cases}
\]

In Equation (9), \( A_c = \frac{dW_m}{2d} \) is the amplitude of functions for coarse measurement. \( \phi_c \) is the initial phase of displacement, which is related to electrode arrangement and displacement starting position.

The schematic curves of functions \( C_A(x)-C_B(x) \) for fine measurement are shown in Figure 1b, and schematic curves of functions \( C_E(x)-C_H(x) \) for coarse measurement are shown in Figure 1d. It can be seen that the two curves in each pair \( C_A(x) \) and \( C_B(x) \), \( C_C(x) \) and \( C_D(x) \), \( C_F(x) \) and \( C_P(x) \), \( C_G(x) \) and \( C_H(x) \) have opposite phase. It also demonstrates that \( C_f(x + \phi_f) \) and \( C_f(x - T_f/4 + \phi_f) \) are orthogonal to each other and \( C_c(x + \phi_c) \) and \( C_c(x - T_c/4 + \phi_c) \) are orthogonal.

3. Modification of Measurement Signal Model

In practical applications, capacitance signals generated by measuring system slightly deviate from theoretical value due to many factors including fabrication errors, installation errors and environment disturbance. In Figure 2, some types of fabrication errors (Figure 2a, Figure 2b, Figure 2c) and installation errors (Figure 2d, Figure 2e) are illustrated. Figure 2a shows the dimension and position errors of electrodes, and these fabrication errors may be caused by accuracy limitations of processing methods and equipment. Signal orthogonality will be changed by position error \( e_1 \).

Ununiformity of spacing between two adjacent electrodes is expressed by position error \( e_2 \), and periodic consistency of signals is influenced by this error. Size error of electrodes is remarked by \( e_3 \), and this error will lead to amplitude variation with displacement change in periodic capacitance signals. Although the dielectric surfaces of MOVER and STATOR are always kept in contact when they relatively slide, the gap \( d \) still varies slightly because of some factors, such as ununiformity of dielectric film thickness and deformation of substrate. The thickness of dielectric film coated on electrodes is not uniform due to the limitation of processing method, and such a case is shown in Figure 2b. Another case in Figure 2c is the deformation of substrate maybe caused by internal stress or external mechanical force. Variation of gap \( d \) will result in not only amplitude change in the same periodic signal but also amplitude difference among different signals, and this will also make signals nonlinear. Figure 2d shows that there is a rotation angle between MOVER and STATOR, and such installation error will result in amplitude difference and phase error in different measurement signals.

Misalignment error between MOVER and STATOR is illustrated in Figure 2e, and this installation error can be avoided by making the length \( L_{fg} \) of electrodes on MOVER larger than the length \( L_I \) of those on STATOR. Besides fabrication errors and installation errors, environment disturbance, such as mechanical vibration and electromagnetic interference, also has an impact on measurement signals.

The above analysis shows that fabrication errors, installation errors and environment disturbance bring measurement signals with problems of amplitude change, non-orthogonality, nonlinearity and noise. Thus, the measurement signal model can be modified:

\[
C'_p = C''_p f_p + N, \quad p = A, B, ... H.
\]

In Equation (10), \( C''_p \) (\( p = A, B, ... H \)) are measurement signal functions modified with non-orthogonality: \( f_A \sim f_H \), which are called scaling coefficient functions, are introduced to modify signal model due to amplitude change and nonlinearity. \( N \) is noise added to measuring signal model and it represents \( N_f^m + N_f \) (\( p=A,B,C,D \)) or \( N_c^m + N_c \) (\( p=E,F,G,H \)). Due to interference from electromagnetic signal and mechanical vibration, \( N_f^m \) and \( N_c^m \) are high frequency noises introduced into signals for fine measurement and coarse measurement. \( N_f \) and \( N_c \) are white noises also added to signals for
fine measurement and coarse measurement. (More information can be seen in supplementary materials).

Figure 2. Illustrations for different error types of fabrication and installation. (a) Dimension and position errors of electrodes; (b) Ununiformity of dielectric film thickness; (c) Substrate deformation; (d) Rotation error between MOVER and STATOR. (e) Misalignment error between MOVER and STATOR.

Figure 3. Simulation curves of signal functions before and after modification.

Measuring signal model is built according to Equation (10) and Figure 3 illustrates the simulation curves of signal functions before and after modification.

4. Signal Processing Method

To reduce noise and increase sensitivity and SNR, signal processing methods are investigated. Based on the measuring signal model established above (Equation (10)), three signal processing methods, including differential, ratio and differential-ratio, are analyzed. Definition of signal processing methods can be seen from Table 1 and simulated curves of fine signals and coarse signals are shown in Figure 4. In each method, maximum variable signal, noise and displacement resolution are discussed in detail (more information can be seen in supplementary materials).
Table 1. Definition of signal processing methods

<table>
<thead>
<tr>
<th></th>
<th>Differential method</th>
<th>Ratio method</th>
<th>Differential-ratio method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fine signals</strong></td>
<td>$y_{f1}^d = C_A' - C_B'$</td>
<td>$y_{f1}^r = C_A'/C_B'$</td>
<td>$y_{f1}^{dr} = C_A'C_B'/C_A'C_B'$</td>
</tr>
<tr>
<td></td>
<td>$y_{f2}^d = C_C' - C_D'$</td>
<td>$y_{f2}^r = C_C'/C_D'$</td>
<td>$y_{f2}^{dr} = C_C'C_D'/C_C'C_D'$</td>
</tr>
<tr>
<td><strong>Coarse signals</strong></td>
<td>$y_{c1}^d = C_E' - C_F'$</td>
<td>$y_{c1}^r = C_E'/C_F'$</td>
<td>$y_{c1}^{dr} = C_E'C_F'/C_E'C_F'$</td>
</tr>
<tr>
<td></td>
<td>$y_{c2}^d = C_G' - C_H'$</td>
<td>$y_{c2}^r = C_G'/C_H'$</td>
<td>$y_{c2}^{dr} = C_G'C_H'/C_G'C_H'$</td>
</tr>
</tbody>
</table>

Figure 4. Simulated curves of fine signals and coarse signals obtained by three different processing methods. (a) Curves obtained by differential; (b) Curves obtained by ratio; (c) Curves obtained by differential-ratio.

As shown in Table 2, simulation results from three signal processing methods are compared. Signal resolutions of both fine signals and coarse signals are significantly improved by signal processing. Signal resolutions from differential method are much higher than those of the other two methods. Signal resolutions of fine signals processed with ratio method are 6-7 times worse than those of differential method and coarse signals' resolutions are 3-4 times worse. Signal resolutions from ratio and differential-ratio are in the same order. As demonstrated in Figure 4b, simulated
curves based on ratio algorithm are obviously distorted and lost their original characteristic. When $N_f^m = 0$ and $N_c^m = 0$, signal resolutions are recalculated, and those according to differential and differential-ratio algorithm are almost the same (Table 3).

<table>
<thead>
<tr>
<th>method</th>
<th>$\eta_{y_{f1}}$</th>
<th>$\eta_{y_{f2}}$</th>
<th>$\eta_{y_{c1}}$</th>
<th>$\eta_{y_{c2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>differential</td>
<td>17116</td>
<td>16896</td>
<td>5612</td>
<td>5644</td>
</tr>
<tr>
<td>differential-ratio</td>
<td>16180</td>
<td>16244</td>
<td>5308</td>
<td>5324</td>
</tr>
</tbody>
</table>

5. Displacement Calculation Model

From Figure 4, we can see that there are regions where signals are insensitive to displacement change at the peaks and troughs of both fine signals and coarse signals and these regions called dead zone can’t be used for measurement. The two fine signal curves are orthogonal to each other and the two coarse signal curves are orthogonal, too. By selectively and alternately using the two orthogonal signal curves, the dead zone regions can be avoided for the measurement. Displacement calculation model includes a coarse calculation model and a fine calculation model. Displacement calculated by coarse calculation model provides absolute position information. With the assistance of absolute positon information, fine calculation model is used to further calculate high-resolution displacement through two fine signals. Displacement calculation model can be denoted:

$$x = f(y_{c1}, y_{c2}, y_{f1}, y_{f2})$$

(11)

Take simulation curves of coarse signals by differential method as an example to analysis. As illustrated in Figure 5, parameters $y_{o1}, y_{o2}$ and $y_i$ are determined to help segment coarse signal curves and build coarse calculation model [16]. As shown in Figure 5a, point A and point B are intersection points of two coarse signal curves. The $y = y_o$ reference line is selected under the condition of $y_{c2}(x = 0) \leq y_o \leq y_{c2}(x = L)$. The $y = y_L$ reference line is selected in the case of $\max (\min (y_{c1}), \min (y_{c2})) < y_L < y_{c2}(x = x_d)$. The $y = y_H$ reference line is chosen according to $y_{c2}(x = x_d) < y_H < \min (\max (y_{c1}), \max (y_{c2}))$. Coarse signal curves are divided into five linear parts used to calculate coarse displacement: DD', EH, JK, NO, RQ. There is an overlap interval between two adjacent linear parts to ensure the reliability of calculation, and one of the overlap is marked in Figure 5a. As shown in Figure 5b, when $y_{c1} \geq y_H$ and $y_{c2} \leq y_o$, the linear part DD' of $y_{c2}$ is fitted into function $f_{DD'}(y_{c1}, y_{c2})$ to calculate coarse displacement ($x_c$). As shown in Figure 5c, under the condition of $y_L \leq y_{c1} < y_H$ and $y_{c2} < 0$, the linear part EH of $y_{c1}$ is fitted into function $f_{EH}(y_{c1}, y_{c2})$ to compute coarse displacement. As demonstrated in Figure 5d, when $y_{c1} < y_L$, the linear part JK of $y_{c2}$ is fitted into function $f_{JK}(y_{c1}, y_{c2})$ to determine coarse displacement. As illustrated in Figure 5e, under the condition of $y_L \leq y_{c1} < y_H$ and $y_{c2} > 0$, the linear part NO of $y_{c1}$ is fitted into function...
(12)
Every linear part is expressed by polynomial function labeled with $f_i^c(y_{f1}, y_{f2})$ and fine calculation model is described as

$$x_f = f_n^c(y_{f1}, y_{f2}), n=1, 2, \ldots$$  \hspace{1cm} (13)

where, $n$ is identification number, which is obtained by:

$$n = \left\lceil \frac{x_c + T_f/4 - x_0}{T_f/4} \right\rceil$$ \hspace{1cm} (14)

In Equation (14), symbol $\lceil \cdot \rceil$ is an up rounding operator, and $x_0$ is a constant representing displacement initial phase of fine signal curves. According to Equation (11) ~ Equation (14), displacement calculation model is eventually expressed:

$$x = f(y_{c1}, y_{c2}, y_{f1}, y_{f2}) = \frac{f(x)_{c1}(y_{c1}, y_{c2}) + T_f/4 - x_0}{T_f/4} (y_{f1}, y_{f2})$$ \hspace{1cm} (15)

In Equation (15), when $y_{c1} \geq y_H$ and $y_{c2} \leq y_0, i=1$; when $y_L \leq y_{c1} < y_H$ and $y_{c2} < 0, i=2$; when $y_{c1} < y_L, i=3$; when $y_L \leq y_{c1} < y_H$ and $y_{c2} > 0, i=4$; when $y_{c1} \geq y_H$ and $y_{c2} \geq y_0, i=5$.

![Figure 6. Schematic diagram of identification number for fine calculation model.](image)

6. Analysis of Error and Accuracy

This section mainly deals with the analysis of error and precision based on displacement calculation model. Within the displacement range, for any displacement position $x^0$, there are corresponding signals $y^0_{f1}$, $y^0_{f2}$, $y^0_{c1}$, $y^0_{c2}$, and identification number $n^0$. According to Equation (12), Equation (14) and Equation (13), coarse displacement, identification number and fine displacement are respectively calculated:

$$x^0_{c_{cal}} = f_i^c(y^0_{c1}, y^0_{c2})$$ \hspace{1cm} (16)

$$n^0_{c_{cal}} = \left\lceil \frac{x^0_{c_{cal}} + T_f/4 - x_0}{T_f/4} \right\rceil$$ \hspace{1cm} (17)

$$x^0_{f_{cal}} = f_i^f(y^0_{f1}, y^0_{f2})$$ \hspace{1cm} (18)

The final error of model is

$$\delta = x^0 - x^0_{f_{cal}}$$ \hspace{1cm} (19)

According to Equation (17), the value of $(n^0 - n^0_{c_{cal}})$ can be -1, 0 or 1 due to coarse displacement error $\Delta x^0 = x^0 - x^0_{c_{cal}}$. When $n^0 - n^0_{c_{cal}} = 0$, there is $\delta = x^0 - f^f_n(y^0_{f1}, y^0_{f2})$. In this case, signal $y^0_{f1}$ is
just in linear part of curve $y^d_{f1}$ and displacement is calculated by fine calculation function $f^f_{n0}(y_{f1}, y_{f2})$ (Figure 7b). Thus, error $\delta$ is determined by function $f^f_{n0}(y_{f1}, y_{f2})$ and it can be improved by properly adjusting degree of fine calculation functions. The analysis is the same when $n^0 - n^0_{cal} = 1$ or $n^0 - n^0_{cal} = -1$. The case of $n^0_{cal} = n^0 - 1$ is shown in Figure 7c. In this case, fine displacement should be calculated by fine function $f^f_{n0}(y_{f1}, y_{f2})$ using signal $y^0_{f1}$, but it is actually calculated by $f^f_{n0-1}(y^0_{f1}, y^0_{f2})$ using signal $y^0_{f2}$ due to the coarse displacement error $\Delta x^c_{c0}$. However, the error $\delta = x^0 - f^f_{n0-1}(y^0_{f1}, y^0_{f2})$ is usually large because signal $y^0_{f2}$ is beyond the interval range of function $f^f_{n0-1}(y_{f1}, y_{f2})$. At the boundaries of two adjacent fine calculation functions, the bigger the coarse error $\Delta x^c_{c0}$ is, maybe the bigger the error $\delta$ is. To reduce the error $\delta$, we can increase interval length to make sure that signal $y^0_{f2}$ is in the “internal” interval of fine calculation function $f^f_{n0-1}(y_{f1}, y_{f2})$. After expanding interval range from $T_f/4$ to $T_f(1 + 2\varepsilon)/4$, coarse error $\Delta x^c_{c0}$ should satisfy $|\Delta x^c_{c0}| \ll \varepsilon T_f/4$. But it is important to note that $\varepsilon$ should be as small as possible to avoid much more nonlinearity in fitting section of fine calculation functions.

![Figure 7](image_url)

**Figure 7.** Schematic diagram of error and accuracy analysis of displacement calculation model. (a) Schematic diagram of signal processed by method of differential; (b) Schematic diagram of error and accuracy analysis in the condition of $n^0 - n^0_{cal} = 0$; (c) Schematic diagram of error and accuracy analysis in the condition of $n^0 - n^0_{cal} = 1$.

It can be known that the error of calculation model is determined by the error of fine calculation functions and slightly affected by the error of coarse calculation functions. The fine displacement model is composed of many functions, and the errors of each function do not affect each other. According to displacement calculation method and error analysis, displacement accuracy is not affected by size accuracy of electrodes. It also can be seen that signal amplitude difference, non-orthogonality and signal offset are also not related to displacement accuracy.

### 7. Experiments and Discussions

To verify the sensing principle, a prototype has been fabricated. The MOVER and STATOR of the measuring system are fabricated by micro machining method. A kind of wafer glass called BF33 is selected as the substrate and gold is chosen as electrode material deposited on the glass substrate.
The periodical width of fine grating groups located in the center is 400 µm with \( W_f \) of 160 µm and \( I_f \) of 40 µm. That of coarse grating groups on the sides is 9.9 mm with \( W_c \) of 3.96 mm and \( I_c \) of 0.99 mm. A layer of Si3N4 with thickness about 500 nm is sputtered on the surfaces of electrodes as dielectric film, owing to its excellent properties, such as significant thermal stability, corrosion resistance, low density, high hardness and low friction coefficient [51-54]. The MOVER can move relative to STATOR in a contact mode with the dielectric film.

Figure 8 shows the results of adhesion strength between Si3N4 and base material using Nano-Scratch tester. Adhesion strength is expressed as the normal critical load when film of Si3N4 is just exfoliated and broken from base material. It can be seen from Figure 8a and Table 4 that the adhesion strength between Si3N4 and BF33 substrate is about 51.48 mN. The value is about 14.66 mN between Si3N4 and gold material, which is shown in Figure 8b and Table 4. The adhesion strengths can meet the demand of use.

![Figure 8](image.png)

**Figure 8. Adhesion strength between Si3N4 and base material using Nano-Scratch tester.** (a) Adhesion strength between Si3N4 and BF33 substrate; (b) Adhesion strength between Si3N4 and gold material.

| Table 4. Adhesion strength between Si3N4 and base material |
|-----------------|-------|------|--------|-------|
| Base material   | 1     | 2    | 3      | Average value |
| BF33            | 51.79 | 50.58| 52.06  | 51.48 |
| Au              | 16.29 | 15.01| 12.68  | 14.66 |

As shown in Figure 9, the planarity of MOVER and STATOR is obtained by using zygo Nexview™ white-light interferometer. The three-dimensional surface morphology of STATOR and MOVER can be seen from the white light interference map (Figure 9a and Figure 9c). As illustrated in Figure 9b, it can be seen that height difference of the STATOR substrate is only 0.533 um over the span of 17 mm and thus the planarity is about 0.031‰. It also can be seen from Figure 9d that height difference of the MOVER substrate is 0.629 um over the span of 13 mm and thus the planularity is about 0.048‰. The deformation of substrate may be caused by high temperature and internal stress during manufacturing process. During MOVER moves relative to STATOR in a contact mode, there is a very small gap change caused by substrate deformation. As mentioned above, scaling coefficient functions are introduced to modify gap change in measuring signal model.

Experiments are carried out to calibrate displacement measuring system, assist to establish displacement calculation model and test measuring system performance. The overall experimental setup is illustrated in Figure 10a and much more details are shown in Figure 10b. The STATOR and MOVER are mounted on mechanical parts to guide the MOVER to move relative to the STATOR in a contact mode. The pushrod mounted on high accuracy (< ± 100 nm) motorized positioning system drives the MOVER to move relative to the STATOR. HEIDENHAIN-CERTO length gauge is used to calibrate the displacement measuring system. Signal acquisition system can collect displacement measuring system signals at a maximum sampling rate about 3600 Hz.
Figure 9. Planarity of MOVER and STATOR is obtained by using zygo Nexview™ white-light interferometer. (a) The three-dimensional surface morphology of STATOR; (b) Height difference of the STATOR substrate; (c) The three-dimensional surface morphology of MOVER; (d) Height difference of the MOVER substrate.

Figure 10. (a) Photograph of experimental setup; (b) Displacement measuring system in detail.

Figure 11a shows the experimental signal-displacement curves of displacement measuring system with range of 5 mm. The parasitic capacitance of four signals for fine measurement is about 43000 units and the amplitude $A_f$ is about 21000 units. Signal amplitude difference and variation of signal amplitude are also reflected in the envelope lines. It can be seen that the four signals for coarse measurement have parasitic capacitance about 30000 units and amplitude $A_c$ of about 21500 units.
According to experimental graphs of fine signals and coarse signals from three different signal processing methods (Figure 11b-d), the same conclusion as theoretical analysis can be drawn that only curves obtained from ratio method are obviously distorted and lose their original characteristic. Three aspects including maximum variable signal, signal noise and signal resolution are listed in Table 5 showing the differences of three signal processing methods. Resolutions of fine signals are about 2-3 times higher than those of original signals, but resolutions of coarse signals have no much improvement. Although the two fine signals’ resolutions from ratio are improved, the ratio method still can’t be applied due to the distortion and bend. There are not so much differences in fine signals’ resolution between differential and differential-ratio methods perhaps accounting for the low intensity high frequency noise. Displacement resolutions from differential and differential-ratio methods are about 4.8 nm and 5.9 nm, respectively.

From Figure 11b or Figure 11d, it can be seen that amplitudes of both two fine signals slightly change with displacement, and there is also a little difference in amplitude between two fine signals. The two fine signals are not orthogonal with each other due to small phase errors, and there also exist DC errors in signals. However, the error of measuring system is still less than ±40 nm (Figure 11e), and system accuracy is not affected by the amplitude error, phase error and DC error. As illustrated in Figure 11e, there is not much difference in measuring system accuracy by using differential method and differential-ratio method.

![Figure 11. Experimental signal-displacement curves. (a) Original signal-displacement curves; (b) Signal curves obtained from differential method; (c) Signal curves obtained from ratio method; (d) Signal curves obtained from differential-ratio method; (e) Error of measuring system by using differential method and differential-ratio method.](image)

![Table 5. Maximum variable signal, signal noise and signal resolution with different signal processing methods.](table)
As illustrated in Figure 12a, the overall bias of coarse calculation model is ±13.53 μm when the fitting degree is 3 and overlapping interval \( \varepsilon \) is 0.1. The overall bias of fine calculation model with degree of 7 and overlapping interval \( \varepsilon \) of 0.1 is ±31 nm, which can be seen from Figure 12b. Figure 12c shows the calculated displacement through the established displacement calculation model with coarse degree of 3, coarse overlapping interval of 0.1, fine degree of 7 and fine overlapping interval of 0.1. It can be seen that the calculated displacement is of uniqueness in the whole displacement range of 5 mm. As demonstrated in Figure 12d, the accuracy of the displacement calculation model is ±34 nm, which is slightly larger than that of the fine displacement model of ±31 nm.

**Figure 12.** Experimental results of measuring system (a) Accuracy of coarse calculation model with degree of 3 and overlapping interval of 0.1; (b) Accuracy of fine calculation model with degree of 7 and overlapping interval of 0.1; (c) Calculated displacement in the whole displacement range; (d)
Displacement calculation model accuracy with coarse degree of 3, coarse overlapping interval of 0.1, fine degree of 7 and fine overlapping interval of 0.1; (e) Displacement calculation model accuracy with coarse degree of 3, coarse overlapping interval of 0.1, fine degree of 5 and fine overlapping interval of 0.1; (f) Displacement calculation model accuracy with coarse degree of 3, coarse overlapping interval of 0.1, fine degree of 7 and fine overlapping interval of 0.

In order to verify the influence of fine degree on model accuracy, another displacement calculation model, only different in fine degree, is established with the same experimental data. The accuracy of the model with fine degree of 5 is ±60 nm (Figure 12e), nearly twice as much as that in Figure 12d. Displacement calculation model, only different in overlapping interval, is also established to find out the effect of overlapping interval on model accuracy. The overall accuracy ±36 nm is slightly larger than that of ±34 nm, but there are several local points with very large error (Figure 12f).

It can be seen that even if the local maximum bias of coarse model reaches ±17 um (Figure 12a), it does not affect the accuracy of displacement measuring system for tens of nanometers (Figure 12d). The accuracy of measuring system mainly depends on the accuracy of fine model and affected by the accuracy of coarse model. The overall accuracy of measuring system can be improved by properly adjusting the fine degree and fine overlapping interval. The accuracy of measuring system is a little higher than that of fine model maybe due to the fact that it is also affected by the accuracy of coarse model.

8. Conclusions

This work describes a novel displacement measuring system based on capacitive grating, which is capable of obtaining absolute measurement with both high-accuracy and long-range. The contact working mode between the MOVER and STATOR minimizes gap change. Dead zone problem is solved through two orthogonal signals configuration. Two orthogonal periodic coarse signals are used providing absolute displacement information in a simple method, and two orthogonal periodic fine signals are further used determining high accuracy displacement with the help of absolute information.

Several kinds of fabrication errors, installation errors and environment disturbance are analyzed and these errors affect signal amplitude, signal phase and signal offset. Measurement signal model is modified according to the influence on signal amplitude, signal phase and signal offset. Signal processing methods including differential method, ratio method and differential-ratio method have been discussed, which indicates that signal sensitivity and SNR can be effectively improved. Signal curves from ratio method lose their original characteristic. Signal resolutions from differential method and differential-ratio method are improved significantly in comparison with those of signals before processed. Analysis of error and accuracy shows that the overall accuracy of calculation model can be improved by properly adjusting the degree of fine functions and fine overlapping interval. The accuracy of calculation model mainly depends on the accuracy of fine model. However, improper handling of the coarse model can result in large displacement errors in some local positions. According to the displacement calculation method and error analysis, displacement accuracy is not affected by electrode fabrication errors. It also can be drawn that signal amplitude difference, non-orthogonality and signal offset are also not related to displacement accuracy.

Experiments are consist with the theoretical analysis mentioned above. Experiment results show that the adhesion strengths between Si3N4 and base material meet the demand of use. The three-dimensional surface morphology of STATOR and MOVER shows that the substrate has very tiny deformation with planarity less than 0.05%. Conclusions in signal processing method section and analysis of error section are confirmed in experiments. The measuring system with range of 5 mm shows that the displacement resolution and accuracy can reach ±4.8 nm and ±34 nm, respectively. Both experiments and theoretical analyses indicate that measuring system have great potential of both tens of nanometers accuracy and hundreds of millimeters range.
Supplementary Materials: The following are available online at

Author Contributions: Conceptualization, Dongdong Zhang and Li Lin; Data curation, Dongdong Zhang and Li Lin; Formal analysis, Dongdong Zhang; Funding acquisition, Quanshui Zheng; Investigation, Dongdong Zhang; Methodology, Dongdong Zhang; Project administration, Li Lin; Resources, Dongdong Zhang; Supervision, Li Lin and Quanshui Zheng; Validation, Dongdong Zhang; Visualization, Dongdong Zhang; Writing – original draft, Dongdong Zhang; Writing – review & editing, Li Lin and Quanshui Zheng.

Acknowledgments: This work was supported by the National Natural Science Foundation of China (grant numbers 51675304, 11890672).

Conflicts of Interest: The authors declare no conflict of interest.

References


41. Nakajima, Y.; Schibli, T.R.; Xu, B.; Minoshima, K. Absolute distance measurement method with optical frequency comb interferometer based on balanced optical cross correlator and optical heterodyne technique.

In Proceedings of Conference on Lasers and Electro-Optics, San Jose, California, 2016/06/05; p. SM2H1.1.


© 2019 by the authors. Submitted for possible open access publication under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).