Comments and Suggestions for Authors

This is a revision of a paper that deals with a conjugate gradient root-finding method for monotone nonlinear systems. Moderate/major revisions were required by several reviewers in the first round. Unfortunately, I do not think that comments have been well addressed and that the paper has been sufficiently improved for publication.

Comment 1: Reviewer 1 asked for an algorithm to compute the regularizers, however, this is vaguely commented in the response letter and I do not see how this is addressed in the paper at all.

Response: Reviewer 1 mentioned in his comments that we introduced the regularization parameters $\mu$, $\sigma$, $\rho$. However, we are not the first to introduce such parameters. The parameters $\sigma$ and $\rho$ were introduced by Solodov and Svaiter [1]. As for $\mu$, to the best of our knowledge, it was first introduced by Yuan and Zhang [3] and we have give reasons for choosing $\mu > 0$. Basically, there is no any algorithm for choosing the parameters. What is done is to keep changing the values of the parameters (within their domain) and running the algorithm until you get the best parameter for your algorithm. If there is any algorithm for choosing the parameters, we are not aware of such algorithm for now.

Comment 2: Another criticism that has not been sufficiently addressed is the numerical accuracy. The results still use $10^{-5}$ accuracy that is said, arguably enough, that is the standard threshold (also used in some other papers). I strongly disagree at this point and insist on an example showing performance of the algorithm for the double float ($10^{-16}$) accuracy. The standard computational error in most of the C/C++ codes is double float, not $10^{-5}$.

Response: An example showing the performance of the algorithm for double float ($10^{-16}$) accuracy was presented in Table 9 as suggested by the reviewer.

Comment 3: Several relevant papers on global non-linear solvers have been suggested, but all have been completely ignored. These subdivision-based solvers are *global* methods that guarantee to find *all* roots inside a domain and within a very fine double-float accuracy, while the proposed approach is only a *local* method that looks only for the closest root. This fact should be clearly discussed in the introduction and a proper link to subdivision solvers should be made.

Response: We have clearly discussed the suggestion by the reviewer in the introduction together
with a proper link to subdivision solvers.

**Comment 4**: Another point that has not been clarified is the monotonicity of the system (1), namely how this very restrictive assumption is validated in all the examples on image processing. Are all the systems monotone? And if not, what is the limitation of the method? These questions have not been answered. Overall, I think that the revision has not addressed the issues raised in the first round and the paper needs another round of revision before being considered for publication again.

Response: Not all systems are monotone (response to Are all the systems monotone?) but our algorithm can only handle those that are monotone (response to what is the limitation of the method?). We have mentioned (in the introducing section) that our proposed algorithm deals with monotone nonlinear equations, furthermore, in subsection 4.1 of the manuscript, we adopted the idea by [2] and transformed the $\ell_1$ regularization problems to equivalent nonlinear monotone equation so that our proposed algorithm can handle such kind of problems.

Finally, we will to thank the anonymous reviewer for his valuable comments which help in improving the manuscript.

**References**

