A Model Predictive Water-Level Difference Control Method for Automatic Control of Irrigation Canals

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Abstract: Previous scholars have proposed and used a water level difference control strategy that works to keep relative deviations in all pools the same using the centralized linear quadratic regulator (LQR) control method, based on a situation in which the operator does not have full control over the canal inflow. In practice, the deviation tolerance of pools may differ in some canals, which limits the applicability of the control strategy. In this work, a weight coefficient was added to the deviation, and the algorithm was improved, to keep the relative deviations to certain proportions. The model predictive control (MPC) method was then used with this improved control strategy and compared to the LQR control method, using the same control strategy. The results showed that the improved strategy can keep the water level deviations in all pools to certain proportions, as is our objective. Also, under this difference control strategy, the MPC method greatly improved performance as compared to the LQR control method, with advanced control actions implemented before delivery changes.

Keywords: Water level difference; MPC control; Weight coefficient; Canal automation

1. Introduction

Irrigation systems are built to deliver large amounts of water from a place with sufficient water to a place where water is a scarce resource. However, a considerable amount of water is wasted due to evaporation, leakage, and lack of control [1]. Innovative and adaptive modernization of irrigation systems is a key factor in improving water use efficiencies.

Improving the operations of irrigation water delivery systems has been an important topic for several decades [2]. Canal automation has evolved to the point where most new canal designs and canal modernization projects include some level of automation [3-5]. Different automatic control methods have been designed, implemented, and developed for canal operation. These automatic control methods can be roughly divided into single-input, single-output (SISO) and multiple-input, multiple-output (MIMO) controllers [6]. In the SISO control method, a single check gate is controlled...
according to a single water level input, such as in the proportional-integral (PI) control method [7] and in improved forms of the PI control method [8, 9]. In the MIMO control method, all check gates are simultaneously controlled according to water level inputs at all monitoring points, such as in the linear quadratic regulator (LQR) [10] and model predictive control (MPC) methods [11].

Although the form of these controllers differ, they all aim to maintain the downstream water level of canal pools at a certain level. Meanwhile, high performance on-field practices require that the water be delivered with sufficient reliability, equity, and flexibility [11]. In some conditions, some control structures may not be fully controlled or cannot be controlled flexibly by an automated system, so these control structures separate the whole canal system into canal segments, with the outflow gate and inflow check gate of each segment fixed. Clemmens et al. [13] proposed another operational strategy to control water levels in long main canals with considerable transmission time or when there is no control on canal inflow and outflow. In this proposal, the water level differences between adjacent pools are the controller inputs, rather than the traditional pool water level deviations. The goal of this control method is to make the water levels in all pools change at the same rate, so that the main canal consequently behaves as a storage reservoir when canal inflow and outflow do not match. Guan et al. [14] applied this control strategy with a centralized LQR to a model of the Central Arizona Project (CAP) main canal, with results showing that the method is a promising way to accommodate mismatches in supply and demand through in-channel storage when there is no full control over the canal inflow. Hashemy et al. [15] used this kind of control system with a simple PI control plus filter (PIF) method to deal with delivery disturbance with fixed head gate. Kong et al. [16] applied this difference control strategy with PI control method to a condition in which the inflow is to be changed significantly.

As the system controls the deviation differences in adjacent pools at the downstream side of the pools, the trend of the control result is that all pool deviations tend to be the same. While under some conditions, for example, if the offtake of the pool delivered significantly more water to more important users than to other pools, the water level disturbance of the pool should be less than that of other pools and the allowable water level deviations of each canal pool could be different. This requires the water levels to change at different rates, but not chaotically. In this paper, the water level difference control strategy is improved to meet this demand. Weight coefficients are added to water level deviations, and the control target is not the actual water level difference between adjacent pools, but the difference between water level deviations, amplified via the weight coefficient. Also, as a centralized control method will be used here, it’s not just a simple input change — the related parameters also change. A model predictive control (MPC) method can in general take into account future water delivery disturbances and take action before those disturbances occur when there is a scheduled delivery change, in contrast to the LQR method used in traditional water deviation control [17]. Therefore, the MPC control is used in the simulation discussed in this paper, together with the water level difference control. The result of this method is also later compared to LQR method results.

2. Materials and Methods

2.1. Test canal and scenario

To demonstrate the performance of the proposed water level difference controller, a simulation model of the last six pools of the Middle Route Project (MRP) for the South-to-North Water Transfer Project was built for simulation study. The six pools together were treated as an independent canal in this simulation, with a constant upstream water level boundary and an uncontrolled head gate — which is not the actual condition, but suitable for application of the proposed control method. The total length of this canal system is about 112 km. The initial flow of the head gate is 94.5 m$^3$/s, and outflow of the last gate is 35 m$^3$/s. As the last pool delivers much more flow to a larger city, the disturbance of the last pool should be less than that of other pools. Thus, the weight coefficient of the last pool should be larger than that of the other pools. The construction process of the canal simulation model is not described here, as it is not the focus. And the model tuning is a general problem for all deterministic models [18] and former study has shown that automatic control methods have a
good robustness [14, 19, 20], the model tuning also is not discussed in this paper. The downstream boundary was set as a constant water level boundary, with a water depth of 3 m. The upstream boundary was similarly set as a constant water level boundary, with a water depth of 7 m. The layout is shown in Fig. 1, and the basic parameters and initial flow condition of each the pools are shown in Table 1. The simulation time step for the hydraulic model of the test canal was 1 min here, and the control time step was 10 min.

The six pools are denoted as Pool 1、Pool 2、Pool 3、Pool 4、Pool 5, and Pool 6. The offtake flow in pool i is denoted as d(i), the inflow gate of pool i is denoted as Gate i-1, and the outflow gate is Gate i. The water level in pool i refers to the water level at the downstream end of pool i — which is also the water level immediately upstream from Gate i — as it is often the water level at the downstream end that is of concern, where the pool is at its maximum water depth.

Table 1. Basic parameters and initial flow condition of each pool

<table>
<thead>
<tr>
<th>Pool</th>
<th>Pool length(km)</th>
<th>Bottom width (m)</th>
<th>Side slope</th>
<th>Downstream initial flows(m^3/s)</th>
<th>Offtake initial flows(m^3/s)</th>
<th>Target water depth(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heading</td>
<td>94.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>26.6</td>
<td>21</td>
<td>2</td>
<td>0.00004</td>
<td>87</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>9.7</td>
<td>22.5</td>
<td>2.75</td>
<td>0.00004</td>
<td>70</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>14.9</td>
<td>17</td>
<td>1</td>
<td>0.00004</td>
<td>42</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>20.8</td>
<td>10</td>
<td>2</td>
<td>0.00004</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>14.7</td>
<td>7.5</td>
<td>2.5</td>
<td>0.00004</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>25.4</td>
<td>7.5</td>
<td>2.5</td>
<td>0.00004</td>
<td>35</td>
<td>7</td>
</tr>
</tbody>
</table>

Note: Each pool is composed of many sections. Parameters of the bottom width and side slope are approximate numbers.

The focus of this paper is to obtain a proper distribution of flow imbalance, as people want when there is limited water supply at the head source. Therefore, different water level difference demand conditions were set. In test scenario No. 1, the offtake flow in Pool 6 increased by 5 m^3/s, scheduled at time 10 h; the water level deviation tolerance of all pools was the same; and the deviation weight coefficients were the same. In test scenario No. 2, there was the same offtake change condition as in scenario No. 1, but the water level deviation tolerance of pools were different. Specifically, the water level deviation tolerance of pools Nos. 1-5 was similar, while the tolerance of pool No. 6 was smaller, so the deviation weights coefficient of pool Nos. 1-5 were set to be 1, with the last pool set at 2.

Also, to check the universality of MPC control under this system, another offtake change condition was also tested. In test scenario No. 3, the weight coefficients were the same as in test scenario No. 2, but the offtake flow in Pool 1 was changed.

2.2. Water level difference control strategies

In cases where the inflow is fully controlled, the inflow can be flexibly adjusted to maintain constant water levels in the downstream pools. And for downstream water level control, a check gate
can be adjusted on the basis of the water level deviation at the downstream end of the next pool
downstream. The water level deviation $e_j$ is defined as

$$e_j = y_j - SP_j$$  \hspace{1cm} (1)$$

where $y_j$ is the water level at the downstream end of pool $j$, and $SP_j$ is the water level set point. In
this kind of control strategy, if the pool deviation tolerances differ and people want to keep water
level deviations in some pool small, it can be more easily done by setting the corresponding water
level weight coefficients matrix $Q$ in MPC control method to larger value [21].

In cases where the inflow cannot be controlled, when outflow or offtake flow changes, the
downstream pools should behave as storage reservoirs. Control actions are determined on the basis
of the difference in water level deviation. Deviation difference $D_j$ is defined as

$$D_j = e_j - e_{j+1}$$  \hspace{1cm} (2)$$

In this instance, if people want to keep water level deviations in some pool to a certain
proportion, it is not feasible to merely set the corresponding weight coefficients in matrix $Q$ in MPC
differently. Since it is the water level difference that is controlled here, changing the weight
coefficients in matrix $Q$ would change the water level difference, not the water level deviation. So in
order to control the water level deviation, a weight coefficient is added to it. $D_j$ is redefined as

$$D_j = m_j e_j - m_j e_{j+1}$$  \hspace{1cm} (3)$$

where $m_j$ is the weight coefficient of water level deviations $e_j$, which reflects the relative weight of
the canal pool. Therefore, the $m_j$ of most pools should preferably be set at 1; that of the important
pools, with smaller allowable water deviations, can be set larger.

As it is the redefined $D_j$ that is controlled in this case, the mathematical form of the control
system involving state variables, controlled variables, and control action variables control should be
changed in order to design the proper controller. For control purposes, it is common to use the
integrator-delay (ID) model [22] for canal pools. It assumes that a canal reach is separated into a
uniform flow with the property delay time and a backwater section with the property storage area.

$$h(k + 1) = h(k) + \frac{T_s}{A_s} \left[ q_{in}(k - k_d) - [q_{out}(k) + q_{offtake}(k)] \right]$$  \hspace{1cm} (4)$$

where $h$ is the water level at the downstream end of the pool, $q_{in}(k - k_d)$ is the inflow to the backwater
section with delay time steps $k_d$, $q_{out}(k)$ is the downstream outflow, $q_{offtake}(k)$ is the off-take outflow,
$A_s$ is the average storage area and $T_s$ is the control time step.

As normally the water level deviation is controlled and the flow increment can be more directly
controlled by control structures [23], Eq. (4) is rewritten in an incremental form with water level deviation:

$$e(k + 1) = e(k) + \Delta e(k) + \frac{T_s}{A_s} \left[ \Delta q_{in}(k - k_d) - [\Delta q_{out}(k) + \Delta q_{offtake}(k)] \right]$$  \hspace{1cm} (5)$$

where $\Delta e(k)$ is the increment of $e(k)$ with $\Delta e(k) = e(k) - e(k-1)$; also, $\Delta q_{in}(k - k_d)$, $\Delta q_{out}(k)$ and
$\Delta q_{offtake}(k)$ are the increment of $q_{in}(k - k_d)$, $q_{out}(k)$ and $q_{offtake}(k)$, respectively.

Substituting Eq. (5) into Eq. (3), Eq. (3) is:

$$D_j(k + 1) = D_j(k) + \Delta D_j(k) + m_j T_s \frac{T_s}{A_s} \left[ \Delta q_{in}(k - k_d) - [\Delta q_{out}(k) + \Delta q_{offtake}(k)] \right]$$

$$- m_{i-1} T_s \frac{T_s}{A_{j,i}} \Delta q_{j,i}(k - k_{j,i}) + m_{i+1} T_s \frac{T_s}{A_{j,i+1}} \left[ \Delta q_{i,j+1}(k) + \Delta q_{offtake,i+1}(k) \right]$$  \hspace{1cm} (6)$$
where $\Delta D(k)$ is the increment of $D(k)$. $\Delta q_{i,j}(k)$ is the increment of outflow $q_{i,j}(k)$ of pool $i$. Eq. (6) expresses the relationship between gate flow and the controlled water level difference $D(k)$ of two adjacent pools.

To establish controller design, the ID model of pools should first be determined. The two characteristics of the ID model, delay time and storage area, were calculated by applying the system identification technique \[24, 25\]. The values are shown in Table 2.

### Table 2. Basic parameters and initial flow condition of each pool

<table>
<thead>
<tr>
<th>Pool Characteristics</th>
<th>Pool 1</th>
<th>Pool 2</th>
<th>Pool 3</th>
<th>Pool 4</th>
<th>Pool 5</th>
<th>Pool 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s$ (m$^2$)</td>
<td>582524</td>
<td>441176</td>
<td>327869</td>
<td>447761</td>
<td>361446</td>
<td>431655</td>
</tr>
<tr>
<td>$T_d$ (min)</td>
<td>70</td>
<td>24</td>
<td>35</td>
<td>57</td>
<td>41</td>
<td>75</td>
</tr>
</tbody>
</table>

2.3. Model predictive control

MPC is a control strategy that explicitly uses a simplified process model of the real system to obtain control actions by minimizing an objective function. MPC has three basic components, including a process model, an objective function, and a rolling optimization strategy \[26\]. A process model is used to predict the system output for some time into the future. Normally, a linear invariant state-space model is used as a process model in canal control, with the form

$$x(k+1) = Ax(k) + Bu(k) + Bd(k) \quad (7)$$

$$y(k) = Cx(k) \quad (8)$$

where $x$ is the state vector; $y$ represents the output variables of the modeled water system, which means the $D(k)$ of all pools here; $u$ is the vector of input variables calculated by the controller, which is the $\Delta q(k)$ of the intermediate check gate here; and $d$ is the vector of known measurable disturbances at time step $k$. $A$ represents the system matrix, $B_u$ the input to state matrix, $B_d$ is the disturbance to state matrix, and $C$ is the state to output matrix. As the control time step is 10 min so the delay time steps of the pools are 7, 2, 4, 6, 4, and 8, respectively. Then Eq. (6) was used for controller design to obtain the process model, with $x_{i-1}$ the state vector, $y_{5:1}$ the output vector, $u_{i-1}$ the input vector, $d_{i-1}$ the disturbance vector, $A_{i:1:1}$ the system matrix, $B_{1:1:1}$ the input to state matrix, $B_{1:1:6}$ the disturbance to state matrix, and $C_{5:1:1}$ the state to output matrix. The MPC control and LQR control could then all be designed on this process model.

Then, an output prediction is made using the process model. In the prediction process, the predicted output of the system, $y(k+i|k)$, is determined from the current state vector, $x(k)$, and the future control actions, $u(k+i|k)$. The predicted values for the state and output vectors one time step into the future are expressed as

$$x(k+1|k) = Ax(k) + Bu(k) + Bd(k) \quad (9)$$

$$y(k+1|k) = Cx(k+1|k) = C\left[Ax(k) + Bu(k) + Bd(k)\right] \quad (10)$$

The prediction for the state vector and output vector two time steps into the future are

$$x(k+2|k) = Ax(k+1|k) + Bu(k+1) + Bd(k+1)$$

$$= A\left[Ax(k) + Bu(k) + Bd(k)\right] + Bu(k+1) + Bd(k+1) \quad (11)$$

$$= A^2x(k) + ABu(k) + Bu(k+1) + A^2D(k) + Bd(k+1)$$
\( y(k+2|k) = Cx(k+2|k) \)
\( = C[A^{2}x(k) + ABu(k) + Bu(k+1) + ADd(k) + Dd(k+1)] \) (12)

This process continues until the end of the control horizon, \( m \), is reached, the state vector and output state vector are
\[
\begin{align*}
    x(k+m|k) &= A^{m}x(k) + A^{m-1}Bu(k) + A^{m-2}Bu(k+1) + \ldots + ABu(k+m-2) + Bu(k+m-1) + A^{m-1}Dd(k) + A^{m-2}Dd(k+1) + \ldots + ADd(k+m-2) + Dd(k+m-1) \\
    y(k+m|k) &= Cx(k+m|k) = C[A^{m}x(k) + \sum_{i=1}^{m} A^{m-i}Bu(k+i-1) + \sum_{j=1}^{m} A^{m-j}Dd(k+j-1)]
\end{align*}
\]

After the control horizon has passed, the remaining output predictions are based on the free response only. At the end of the prediction horizon, \( p \), the predicted state vector and output state vector are
\[
\begin{align*}
    x(k+p|k) &= A^{p}x(k) + A^{p-1}Bu(k) + A^{p-2}Bu(k+1) + \ldots + ABu(k+p-2) + Bu(k+p-1) + A^{p-1}Dd(k) + A^{p-2}Dd(k+1) + \ldots + ADd(k+p-2) + Dd(k+p-1) \\
    y(k+p|k) &= Cx(k+p|k) = C[A^{p}x(k) + \sum_{i=1}^{p} A^{p-i}Bu(k+i-1) + \sum_{j=1}^{p} A^{p-j}Dd(k+j-1)]
\end{align*}
\]

An objective function, which is typically a combination of errors of output variables between a given reference and control actions over the prediction horizon, is minimized by adjusting future control actions. It is possible to assume that the output reference is that the \( D(k) \) equals zero. Hence, the objective function can be expressed as
\[
\begin{align*}
    \min_{u(k|k)} J &= \sum_{i=1}^{p} (y^T(k+j|k)Qy(k+j|k)) + \sum_{j=0}^{m} (u^T(k+j|k)Ru(k+j|k))
\end{align*}
\]

where \( p \) is the prediction horizon, \( m \) is the control horizon, which should be less than or equal to \( p \), \( Q \) is the weighting matrix of output, and \( R \) is the weighting matrix of input. The problem can be summarized as minimizing the objective function by adjusting the future control actions \( u(k) \). Once the sequence of future control actions is determined, only the first set of control actions is implemented on the irrigation system. The system is then updated and the process repeated. This is the rolling optimization strategy of an MPC controller.

As an LQR control method is compared later, the background of LQR controllers is also introduced. The LQR method uses the same process model as does the MPC method, but the objective function is an infinite time domain as
\[
\begin{align*}
    \min J &= \sum_{j=0}^{\infty} (y^T(j)Qy(j) + u^T(j)Ru(j))
\end{align*}
\]

For MPC, many times, tuning is done through trial-and-error techniques [27]. There were still four parameters to be determined for MPC control: the prediction horizon \( p \), the control horizon \( m \), the cost weighting matrix \( Q \) for outputs, and the cost weighting matrix \( R \) for the control actions. The prediction horizon, \( p \), should be long enough to include all necessary dynamics of the system. As the total delay time steps of the upstream disturbance on the water level of pool 6 was 24 (sum of the
delay steps of pool 2 to 6), the prediction horizon \( p \) was set to 30 and the control horizon \( m \) was 20. Values within \( Q \) and \( R \) provide a trade-off between minimizing water level differences and minimizing check flow changes, so the matrix \( R \) can be an identity matrix. If \( R \) is relatively greater than \( Q \), the controller will focus more on the minimization of the gate control actions than on water level difference. Therefore, a larger value of \( Q \) is preferred here. Note that a much larger value of \( Q \) may lead to instability in the control system, as the controller is designed on a simplified process model and great changes in the control actions result in great pool response and sometimes great fluctuations and even resonance. The value of the \( Q \) should be chosen carefully and in this simulation, \( Q \) was set as a diagonal matrix with an elements value of 5 for MPC control method.

The two cost weighting matrices \( Q \) and \( R \) were also needed for LQR control. For comparison purposes, the \( Q \) matrix was also a diagonal matrix with an elements value of 5 for LQR control. But as the results of LQR control with this \( Q \) matrix did not perform well in difference control in the Results part, a diagonal matrix \( Q \) with an elements value of 15 was also used in LQR control. The LQR control with matrix \( Q \) with an elements value of 5 is referred to as LQR-I, and the LQR control with matrix \( Q \) with an elements value of 15 is referred to as LQR-II in this paper.

The MPC control method, the LQR-I control method and the LQR-II control method were all used with the difference control strategy for the test scenarios.

3. Results

The simulation results for the test scenarios are presented in Figs. 2-7. Fig. 2 and Fig. 3 are the water level deviation and flow change results for scenario No. 1, respectively. Fig. 4 and Fig. 5 are the water level deviation and flow change results for scenario No. 2, respectively. Fig. 6 and Fig. 7 are the water level deviation and flow change results for scenario No. 3, respectively. Among them, the Fig. 2(a) – 7(a) are the results with MPC method; Fig. 2(b) – 7(b) are the results with LQR-I method; and Fig. 2(c) – 7(c) are the results with LQR-II method. A summary of all simulations is found in Table 3.

In Table 3, several indicators are used: maximum absolute water level deviation (MAE); average absolute water level deviation (AAE); maximum absolute water level difference (MAD); and average absolute water level difference (AAD) and the time gate flow change.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPC</td>
<td>LQR-I</td>
<td>LQR-II</td>
</tr>
<tr>
<td>Pool 1</td>
<td>0.16</td>
<td>0.2</td>
<td>0.19</td>
</tr>
<tr>
<td>Pool 2</td>
<td>0.15</td>
<td>0.19</td>
<td>0.18</td>
</tr>
<tr>
<td>Pool 3</td>
<td>0.14</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Pool 4</td>
<td>0.13</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>Pool 5</td>
<td>0.12</td>
<td>0.1</td>
<td>0.11</td>
</tr>
<tr>
<td>Pool 6</td>
<td>0.1</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Pool 1</td>
<td>0.09</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>Pool 2</td>
<td>0.09</td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td>Pool 3</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>Pool 4</td>
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<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Pool 5</td>
<td>0.07</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Pool 6</td>
<td>0.08</td>
<td>0.07</td>
<td>0.06</td>
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<tr>
<td>Pool 1</td>
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<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Pool 2</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Pool 3</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 3. Summary of simulation results
Fig. 2 and Fig. 3 show the results for scenario No. 1, in which the same coefficient of all pools was used. In Fig. 2, the water levels in all pools tend to decrease at a similar rate with the MPC, LQR-I, and LQR-II methods. But after 20 h, the water level deviations are smaller at the downstream pools compared with those at the upstream using both the MPC and LQR methods. That’s because, though the gates at the both ends are not controlled, the inflow increases and outflow decreases as the water levels decrease. The decrease in outflow is greater than the increase in inflow, so the water level decrease trend is gentle in the downstream pools. As can be seen from Fig. 3, the flow of Gate 0 (the head gate) and Gate 6 (the last gate) also changed, but the flow change of Gate 6 is obviously greater than that of Gate 0.
Figure 2 The simulation water level deviation result for scenario No. 1 with MPC control (a), LQR I (b), and LQR II (c)

Figure 3 The simulation flow change result for scenario No. 1 with MPC control (a), LQR I (b), and LQR II (c)

By comparing the result of Fig. 2(a) with Fig. 2(b) or Fig. 2(c), it can be seen that the MPC control performed better at minimizing the water level deviation difference of adjacent pools, with a maximum value of 0.02 m. The maximum value of the LQR-I method was 0.03 m and that of the LQR-II method was 0.03 m. Consequently, there was a smaller maximum water level deviation in all pools — 0.16 m with the MPC method, compared with 0.2 m with the LQR-I control method and 0.19 m with the LQR-II control method. With a better control in water level difference, the imbalance in flow was better distributed so that the maximum water level deviation caused by flow change among all pools was small. The main advantage of the MPC method was its ability to consider the future disturbance and take action before the disturbance happened. In Fig. 2(a), the gate flow of Gate 1-5 begin to change at the time 6.5 h, 6.6 h, 5.7 h, 5.2 h, 5.5 h. All are ahead of the time 10 h, when the offtake flow changes. While in Fig. 3(b) and Fig. 3(c), the gate flow all begin to change around the time 10 h, only after the disturbance happens. In Fig. 2(a) and Fig. 2(b), the water level in Pool 6 quickly decreases around the time 10 h, while the water level in other pools does not. In Fig. 2(c), the water level in Pools 1-5 have already decreased slowly, and the water level in Pool 6 has increased a little before the time 10 h. After the disturbance happens, the water level in all pools decreases slowly, at nearly the same rate. So forward control actions were taken before the disturbance occurred using the MPC control, and the water level differences were smaller. By comparing Fig. 2(b) and Fig. 2(c), it can be seen that the results show LQR-II was better than LQR-I in minimizing water level difference. That is because in LQR-II, the values of the elements of the weighting matrix Q are greater. But there are also greater water level fluctuations and flow change fluctuations in Fig. 2(c) and Fig. 3(c) as compared with Fig. 2(b) and Fig. 3(b). As greater values in matrix Q may result in significant flow change and water level fluctuations, they are more likely to cause water level resonance, so the values are set at 15 here instead of at a greater value. In a comparison of the results of the MPC and LQR-I...
methods, in which the same matrix Q was used, the MPC performs much better than the LQR-I method. It is not only that advance control actions were taken in this method, but is also that the function J of MPC is to minimize the control actions in control horizon m and the water level differences in prediction horizon p, greater than m here. So more water level differences are considered and the resultant control actions of the MPC method are greater to better minimize the water level differences.

Fig. 4 and Fig. 5 show the results for scenario No. 2, in which the weight coefficient of Pool 6 was set at 2 while the others were set at 1. Fig. 3 shows that with the weight coefficients used, the water level deviation in Pool 6 was smaller than that in the other pools, almost half of the water-level deviation in Pool 5. In the MPC control method, the maximum water level deviation in Pool 6 was 0.07 m, and 0.16 m in Pool 5. Using the LQR-I method, the maximum water level deviations in these two pools were 0.08 m and 0.13 m, respectively, and with the LQR-II method, 0.08 m and 0.14 m, respectively. That means with the proposed strategy, the water level deviations can be a certain proportion as we hope, and by using Eq. (10) to build and design the controllers, the water level and flow control process are reasonable. The water level deviations in Pools 1-5 gradually approached the same value, while the water level in Pool 6 approached another value in both the MPC and LQR methods. Compared with results for scenario No. 1, the water level deviations of Pools 1-5 were much bigger. The maximum water level deviations of Pools 1-5 were 0.2 m, 0.23 m, and 0.23 m with the MPC control method, the LQR-I method, and the LQR-II method in scenario No. 2, larger than those in scenario No. 1 with values of 0.16 m, 0.2 m, and 0.19 m. That’s because the small water level deviation in Pool 6 in scenario No. 2 requires small inflow and outflow imbalance in Pool 6, while the flow imbalance in other pools is greater, so the water levels in these pools decreased more. In Fig. 5(c), the flow fluctuations in Pools 1-5 are much more obvious using the LQR-II method in scenario No. 2 than in scenario No. 1, because the greater flow increase of Gate 5 was required to reduce the decreased rate of the water level in Pool 6 as compared with scenario No. 1. The maximum flow change of Gate 5 was 4.8 m³/s and 5 m³/s in scenario No. 1 and scenario No. 2, respectively.
Figure 4 The simulation water level deviation result for scenario No. 2 with MPC control (a), LQR I (b), and LQR II (c)

Figure 5 The simulation flow change result for scenario No. 2 with MPC control (a), LQR I (b), and LQR II (c)

Fig. 6 and Fig. 7 demonstrate the control results for scenario No. 3, in which the disturbance occurred in Pool 1. Similar to previous results, forward control actions were taken with the MPC method. In Fig. 7(a), the gate flow of Gate 1-5 began to change at time 5.7 h, 5.7 h, 6.2 h, 6.8 h, and 7.8 h, respectively, with the MPC control method, while in Fig. 7(b) and Fig. 7(c) the times were 10.2 h, 10.5 h, 10.5 h, 10.5 h, and 10.5 h, respectively, in the LQR methods. The performance of the MPC in minimizing the water level deviation differences and maximum water level deviation was better than that of the LQR-I and LQR-II methods. The maximum water level deviations of the MPC, LQR-I, and LQR-II methods were 0.18 m, 0.25 m, and 0.22 m, respectively. The maximum water level deviation differences were 0.04 m, 0.06 m, and 0.04 m, respectively. Also, although the water disturbance happened in Pool 1, the water level deviation in Pool 1 decreased quickly when the disturbance happened at time 10 h, but at the time 35 h the water level deviation in Pools 2-5 were closed to water level deviation in Pool 1, and the water level deviation in Pool 6 is about half of that with the MPC control method. While in the LQR control methods, especially with the LQR-I control, the control effects are not obvious, as no forward control actions were taken.
Figure 6 The simulation water level deviation result for scenario No. 2 with MPC control (a), LQR I (b), and LQR II (c).
4. Conclusions

There are several important conclusions that can be made based on the various simulation results. These conclusions are summarized below.

1. Water level difference control allows the operator to have no full control of the head gate and the tailgate while automatically controlling all check gates in between. It reveals flow mismatches by causing the water levels to rise or fall at the same rate.

2. By adding a weight coefficient to the water level deviation to construct water level difference, and with several changes in controller design, the control method can make the water levels rise or fall at different rates in the proportion that people want with flow mismatches, consequently changing the water level deviations with the proportion.

3. Both the LQR and MPC control methods with the proposed control strategy work to minimize the water level difference, but the MPC control performs better when there is a scheduled delivery change. The MPC does so because it takes future disturbances into account and takes control before the offtake flow change to minimize the water level difference.

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