Response to the reviewers’ comments

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- Type of manuscript: Article
- Title: Signal Detection for Ambient Backscatter Communication with OFDM Carriers
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We appreciate your timely review and constructive comments. In the revised manuscript, revisions in the text are displayed in “blue” color. The numbering in the reviewers’ comments and our responses will therefore correspond to the revised version unless explicitly stated.

For the purpose of convenience, we summarize overall organization of the paper as follows.

- In Section 1, we introduce the background knowledge on energy harvesting and the problems of signal detection in AmBC with OFDM carriers.
- In Section 2, we describe the system characteristics and the formulation of the detection problem in terms of spectrum sensing perspective.
- In Section 3, we propose an optimal energy detector. A step-by-step procedure consisting of detection threshold calculation and power order optimization is introduced.
- In Section 4, we discuss analytical and numerical simulation results for the proposed scheme and other conventional approaches.
- In Section 5, we provide conclusions and future directions.

In summary, the following changes have been made in the revised manuscript.

1. Following Comment 1.3 and Comment 2.1, we expanded the discussion on the related works and described the contribution of our paper.
2. We added more descriptions on every figure and modified Fig. 3 in the revised manuscript.
3. Related works have been accordingly updated in References section.
4. Typos and the grammar mistakes have been corrected throughout the entire manuscript.

Also, we would like to ask to assign the revised paper to the same reviewers.
Reviewer 1

Thank you to the authors, the paper and the work are interesting! But they need to be improved.

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Comment 1.1. The abstract should NEVER have any references.

Reply 1.1. We would like to thank the reviewer for the comment. In the original manuscript, we aimed to show the communication system that utilizes AmBC over ambient OFDM signals in [9] in the abstract. In order to avoid this issue, we modify the abstract (line 6-7) as follows.

Second, we take the advantages of the early work on orthogonal frequency division multiplexing (OFDM), where the repeating structure of ambient OFDM signals is exploited to cancel out the direct-link interference by using cyclic prefix, then provide a test statistic in which optimal detection threshold and optimal power order are derived accordingly.

Comment 1.2. The notations should have their own section, not in the end of the Introduction.

Reply 1.2. As the reviewer suggested, in the revised manuscript we make a new section “Notations” in page 2, where the notations are presented separately by their terms.

Comment 1.3. All the sections should have their own short introduction.

Reply 1.3. Following the reviewer’s suggestion, we made several new paragraphs in the revised manuscript. In detail, we add:

- In page 2, Section 2.

  The aim of this section is first to introduce the existing AmBC system that utilizes OFDM carriers, then to present those factors that affect the detector performance. In order to understand how to make use of both energy harvesting and backscattering, we investigate the AmBC system model where an energy harvesting tag uses the existing radio signals from ambient sources to operate itself (e.g., RF source). It produces a modulated reflection of those signals to a nearby receiver (e.g., reader). Legacy receivers employ OFDM structure, where data is transmitted in parallel on a certain number of sub-carriers of different frequencies. Thus, it leads to low power...
consumption on data transmissions which is very suitable for low-powered hardware or batteryless IoT devices. Moreover, in terms of efficient spectrum sensing, it has been shown that energy detection approach has low computational complexity and ability to identify the spectrum holes without a priori knowledge of primary characteristic [14-16]. Once we develop an appropriate test statistic for energy detector, it can guarantee a desired detection performance.

- In page 3, Section 2.2

Note that this tag is equipped with a switch that can split the received signal into two parts: information decoder (ID) and energy harvester (EH). Assume that they are all connected to a single antenna and use the same RF signals. The RF-powered passive tag communicates with the reader by switching its antenna impedance of its backscattered signals. The energy harvester collects the energy from the ambient OFDM signals and uses it to provide a small amount power required for the communication and performing tasks at the tag.

- In page 3, Section 2.3

To ensure the orthogonality of received subcarriers over the useful symbol period as well as efficient joint allocation of subcarriers and powers among legacy users, we need to design a waveform to convey information bit in tag symbol, where the CP is longer than the delay spread of the channel.

- In page 7, Section 4

In the following, we briefly describe some metrics used to evaluate the proposed statistic test. The test must be a sufficient statistic for our energy detection problem and contain all the information required to distinguish two hypotheses $H_0$ and $H_1$.

(i) First, we check the validity of the Gaussian approximation for the proposed test. In fact, since the test statistic (10) follows the Gamma distribution under both hypotheses, the length $D$ must be large enough to apply the CLT while not very large to keep the approximation meaningful. In the first run, Fig. 2 gives us a case study on this approximation.

(ii) Second, in order to find the optimal $p$ for the test (10) instead of using $p = 2$, we solve (25) to obtain an adaptive power order. The result is shown in Fig. 3. The purpose of this result is to observe how the $p^*$ changes for maximizing the probability of correct detection according to the changes in the SNR. Thus, we may have a certain strategy to select $p$ for a given SNR and a false alarm rate.

(iii) Third, with different settings (e.g., SNR and $N_{cp}$), we observe how much the BER changes when using our test statistics and the conventional ones in terms of our ability to solve $p^*$ with high correct detection probability $P_d$. The results are given in Figs. 4 and 5.

(iv) Finally, we provide the median receiver operating characteristic (ROC) curve for our detector design as predicted by our aforementioned analysis.

Comment 1.4. I miss a section (number 2) that covers the state of the art. The authors talk about it briefly in the introduction, but it's not enough for a journal paper. I recommend them to talk about the SotA in an entire section. There is enough literature to have that section. The authors have only 12 cites in this paper, they should increase their literature review.

Reply 1.4. We would like to emphasize that our goal in the original manuscript is to provide a good energy detector design to achieve a better data rate and energy efficiency. Thus, we attempted to show more about how to construct the energy detector as well as estimate the relevant parameters.
According to reviewer’s suggestion, we add more texts in Section 2 as Reply 1.4 presented. The corresponding reference are updated as well. For the purpose of convenience, we give below the numbers of the references mentioned in the reviewer’s comments.


**Comment 1.5.** In Figure 3, I suggest that tests should be done also for SNR=0.5 and 1.5 at least. The change among SNR=0, 1 or 2 is too big to interpolate it without any more tests.

**Reply 1.5.** In the revised manuscript we have investigated the scenario suggested by the reviewer, where $p^*$ value is iteratively evaluated in a smaller SNR step to find the optimal value. Fig. 3 shows the numerical results for this setup.

Moreover, in order to address the reviewer’s comment more clearly, we also added more description in page 9 as follows.

*We also plot a small subgraph at the right hand side of Fig. 3 to illustrate the $p^*$ value (in vertical axis) versus small SNR (in horizontal axis) because we observe that $p^*$-curve has a big jump in its value for $\gamma$ in the range of (0,1.5).*

**Comment 1.6.** Why does the BER remain despite improving the SNR? That question is important and should be answered in the text.

**Reply 1.6.** In the original manuscript we stated that for larger SNR the BER performance remains unchanged. It has been shown that this phenomenon might be caused by the strong direct-link interference at $p=2$ [12]. We now explain why the proposed mechanism and the above result have the same behavior.

- First, as we presented in the original manuscript, since the backscatter signals are weak, the problem of signal detection with small changes needs to be investigated. Thus, it is more interesting in the performance at low SNR levels. For higher or uncertainty noise level, we may investigate other robust schemes to adapt the noise condition [R1]. Thus, the high level of noise, we may concern other approaches to adapt the noise uncertainty.
Figure 3: The optimum value of power order $p$ versus $\gamma$. It shows the effect of $\gamma$ on $p^*$ at several different SNR levels, which comes from the solving procedure of (24).

- Second, in order to use (10), (13) and (14), the reader tries to distinguish two bits by taking a large enough number of samples, i.e., $D$. The value of $D$ we consider here is large enough to apply the central limit theorem, thus the probability of miss detection and the probability of false alarm are moderate, i.e., they are not changed with $D$. The detector must reach the error probabilities uniformly over a whole uncertainty set with various $D$. As the SNR increases, it hits the SNR wall [R1] while the required sample complexity meets our performance targets [R1].


In order to reflect this comment, we added the following texts in page 9 of the revised manuscript.

Moreover, since the reader tries to distinguish between two bits by taking a sufficiently large number of samples, i.e., $D$, the value of $D$ we consider here is large enough to apply the CLT, thus the probability of miss detection and the probability of false alarm are moderate, i.e., they are not changed with $D$. Consequently, the overall BER in (20) does not become much different. The detector must reach the error probabilities uniformly over a whole uncertainty set with various $D$. As SNR increases, it hits the SNR wall [19] while the required sample complexity meets our performance target.

We also added the corresponding paper as [19] in the Reference section of the revised manuscript.

Comment 1.7. The captions of the figures should be more auto-explicative.

Reply 1.7. Thank you for the suggestion. In order to keep the paper consistent, we have carefully checked and revised every figure caption throughout the manuscript. A list of modifications that have been made in the revised manuscript is given below.

- Figure 1. A communication system utilizing the AmBC over OFDM carriers. The AmBC system consists of three main components: RF source (e.g., TV tower), ambient backscatter transmitter (e.g., AmBC tag), and ambient backscatter receiver (e.g., reader), while the legacy OFDM system consists of several legacy receivers (e.g., mobile phones).
• Figure 2. Illustration of CDFs under $H_0$ and $H_1$. Note that the theoretical analysis shows the test statistic $t \sim \Gamma(k_i, \theta_i)$ under $H_i$, while the simulation approximation gives us $t \sim N(\mathbb{E}(t), \text{Var}(t))$, where $\mathbb{E}(t)$ and $\text{Var}(t)$ are given in (13) and (14), respectively.

• Figure 3. The optimum value of power order $p$ versus $\gamma$. It shows the effect of $\gamma$ on $p^*$ at several different SNR levels, which comes from solving procedure of (25).

• Figure 4. BER versus SNR $\gamma$. We observe that BER achieves the maximum at $\gamma = 0$. The designed detector can perform well even when the SNR is high.

• Figure 5. BER versus SNR with $N_{cp}$. As we predicted, the BER increases as $N_{cp}$ decreases.

• Figure 6. ROC curve with SNR $\gamma$. An ROC curve is obtained by taking the average over 100 independent trials.

Comment 1.8. The conclusions are much too short, the authors should conclude wider from their tests.

Reply 1.8. In order to extend this section, we added the following texts in page 10 of the revised manuscript.

Moreover, based on the insightful results we suggest the following directions for future work.

(i) Regarding tag operation, an important direction is to come up with a model that examines the energy harvesting model and enhances the detection performance accordingly.

(ii) In our problem formulation, we use a simple noise uncertainty model, i.e., the variance of $v(n)$ is assumed to be bounded by a given number $B$. This value depends only on a single value $\rho$, thus it may not incorporate the RF strength and other changes in the environments. Therefore, we need to investigate other nonlinear models that relate to energy detector’s inherent noise uncertainty.

(iii) In the problem formulation, we assumed that the tag has two states: backscattering and non-backscattering, while in practice its antenna load may switch among three states: no reflecting, reflecting in the same phase, and reflecting in the opposite phase, resulting in a ternary signal $B(n)$. Thus, we need to design a waveform $x(n)$ to convey the corresponding bits.

We expect that the above future directions can contribute to the advancement of energy detection and estimation areas.

Comment 1.9. The appendix A should have more verbal explanations.

Reply 1.9. In the original manuscript, the appendix A only showed the derivation of solving (25), thus most of the parts were mathematical expressions. In order to reflect this comment, we added more texts in this part as follows.

• In line 217

As we mentioned before, the optimal value $p^*$ can be obtained by simply taking the derivative of $P_d$ and setting it to be zero. The detailed procedure is described as follows.

• In line 219
With other fixed parameters, $P_d$ is a function of the single variable $p$. Thus, we first select an guess interval for $p$, then apply efficient numerical tools (e.g., Newton method [17]) to obtain the approximate roots of (A2). We also assume that the error produced due to computing process can be ignored, i.e., the numerical result is acceptable for all cases.
Reviewer 2

Authors, thank you for your submission. This is a very interesting topic and a thorough analysis will be helpful for other researchers.

Open Review

(x) I would not like to sign my review report

() I would like to sign my review report

() Extensive editing of English language and style required

(x) Moderate English changes required

() English language and style are fine/minor spell check required

() I don’t feel qualified to judge about the English language and style

Does the introduction provide sufficient background and include all relevant references?

Yes (x) Can be improved ( ) Must be improved ( ) Not applicable ( )

Is the research design appropriate?

(x) ( ) ( )

Are the methods adequately described?

( ) ( ) (x)

Are the results clearly presented?

( ) (x) ()

Are the conclusions supported by the results?

( ) (x) ()

Comment 2.1. On page 1, “Second, traditional backscatter receivers are constructed from powered components (e.g. oscillators), while the AmBC ones are battery free.” Could you add more to this statement? Do you envision the AmBC devices communicating with each other, requiring battery free operation, or with a legacy receiver or reader like in Fig. 1, where there is likely a wall connected power source or a battery?

Reply 2.1. Regarding to the first question, it has been shown in [5]-[7] that

- In a traditional backscatter communication system like the RFID system, the RF source generates the RF signals to activate the tag. The tag modulates, reflects those signals, and transmits its data to the reader. Thus, at the reader the backscattered information is decoded by a power-hungry hardware components (e.g., oscillator or analog-to-digital converter).

- In an AmBC system, the tag has two components (information decoder and energy harvester modules) and they are all connected to a single antenna and use the same RF signals as the reader. The tag communicates with the reader by backscattering the ambient signals. The energy harvester collects the energy from the ambient signals and uses it to provide a small amount power required for communication and tag operations. Since the RF signals already exist in the environment, it allows us to avoid such installation and maintenance costs required for the RFID systems.

In the original manuscript (page 1), we explained that the system utilizing the AmBC uses existing RF signals, rather than generating their own radio waves. In order to clarify this point, we added the following sentence in Section 1 of the revised manuscript.

By taking the advantage of existing RF signals in the air, it does not required any additional deployment like the RFID reader that suffers more installation and maintenance costs.

Regarding to the second question, there are two co-existing communication systems in Fig. 1 of the original manuscript.
The legacy OFDM system consists of an RF source (e.g., WiFi, TV tower) and its dedicated legacy receiver (e.g., WiFi client, TV receiver). The RF source transmits OFDM signals to the legacy receivers.

The AmBC system in which consists of a RF-powered passive tag and a single antenna reader. The tag contains a switch, which can split the ambient OFDM signals into two parts: The backscatter modulation modulates its received ambient OFDM carrier, and the RF based energy harvesting module harvest energy from ambient OFDM signals. Thus, the backscattered signal is received and decoded by the reader.

In page 3 of the revised manuscript, we add the following sentences to clarify these comments.

Note that this tag is equipped with a switch that can split the received signal into two parts: information decoder (ID) and energy harvester (EH). Assume that they are all connected to a single antenna and use the same RF signals. The RF-powered passive tag communicates with the reader by switching its antenna impedance of its backscattered signals. The energy harvester collects the energy from the ambient OFDM signals and uses it to provide a small amount power required for the communication and performing tasks at the tag.

Comment 2.2. Page 3, “The tag receives the RF source signal and transmit its modulation signal $c(n)$ to the reader”. It would be more consistent with other backscatter works to say the tag backscatters, reflects, or modulates the RF signal, instead of saying it transmits a modulation signal.

Reply 2.2. Since in the original manuscript the tag modulates its received RF signal $c(n)$ and the carrier signal is $\tilde{c}(n)$. Thus, please understand that the original description is more appropriate for the system model considered in this work.

Comment 2.3. It would be interesting to hear a discussion of the trade off of the tag side requirements for this scheme. The tag is presumably operating from harvested energy so keeping the backscatter modulation rate low or keeping the backscatter time short is required to decrease the RF input power requirement for the tag. How does the CP length impact the tag’s power requirement or data rate?

Reply 2.3. As shown in the original manuscript, the relationship between the tag rate and the CP length is given by

$$R_{\text{tag}} = \frac{f_s}{(N + N_{\text{cp}})}.$$ (27)

Obviously, if we fix the number of OFDM carriers $N$, the data rate $R_{\text{tag}}$ decreases as the CP length increases, while the BER decreases, as illustrated in Fig. 5. This makes a trade-off between the BER and the data rate. We modified this part in the revised manuscript as follows.

However, the smaller $N_{\text{cp}}$ offers higher data rate from the relationship

$$R_{\text{tag}} = \frac{f_s}{(N + N_{\text{cp}})},$$ (27)

where $R_{\text{tag}}$ is the tag rate [12]. Obviously, if we fix the number of OFDM carriers $N$, $R_{\text{tag}}$ decreases as the CP length increases, while the BER decreases, as illustrated in Fig. 5. Thus, there exists a trade-off between the BER and the data rate $R_{\text{tag}}$.

Comment 2.4. Please spend a little more time describing what Fig. 2 is showing. It’s clear from the written description that the theory and simulation agree well but there isn’t much description of what the plots mean.

Reply 2.4. According to the reviewer’ comment, we add more explanations in page 7 of the revised manuscript.
In the following, we briefly describe some metrics used to evaluate the proposed statistic test. The test must be a sufficient statistic for our energy detection problem and contain all the information required to distinguish two hypotheses $H_0$ and $H_1$.

(i) First, we check the validity of the Gaussian approximation for the proposed test. In fact, since the test statistic (10) follows the Gamma distribution under both hypotheses, the length $D$ must be large enough to apply the CLT while not very large to keep the approximation meaningful. In the first run, Fig. 2 gives us a case study on this approximation.

(ii) Second, in order to find the optimal $p$ for the test (10) instead of using $p=2$, we solve (25) to obtain an adaptive power order. The result is shown in Fig. 3. The purpose of this result is to observe how the $p^*$ changes for maximizing the probability of correct detection according to the changes in the SNR. Thus, we may have a certain strategy to select $p$ for a given SNR and a false alarm rate.

(iii) Third, with different settings (e.g., SNR and $N_{cp}$), we observe how much the BER changes when using our test statistics and the conventional ones in terms of our ability to solve $p^*$ with high correct detection probability $P_d$. The results are given in Figs. 4 and 5.

(iv) Finally, we provide the median receiver operating characteristic (ROC) curve for our detector design as predicted by our aforementioned analysis.

Regarding to Fig. 2, we add more description in its captions as

**Figure 2. Illustration of CDFs under $H_0$ and $H_1$.** Note that the theoretical analysis shows that test statistic $t \sim \Gamma(k_i, \theta_i)$ under $H_i$, while the simulation approximation gives us $t \sim N(\mathbb{E}(t), \text{Var}(t))$, where $\mathbb{E}(t)$ and $\text{Var}(t)$ are given in (13) and (14), respectively.

**Comment 2.5.** Why are the simulation parameters in Table 1 used? How would this compare to a common OFDM carrier source?

**Reply 2.5.** In the original manuscript, we used the simulation parameters in Table 1 for generating signals in Section 2. In particular, we set up

- A passband signal $\tilde{s}(n) = \text{Re}\left\{\sqrt{P_s}s(n)\exp\{j\omega t f_c^\frac{n}{T}\}\right\}$, where $f_s = 20$ MHz.

- The maximum delay of multipath channel $L \triangleq \max\{N_{sr}, N_{st} + N_{tr} - 1\}$. From given $N_{cp}$ and $L$, we can calculate the length of samples $D = N_{cp} - L + 1$.

- Given the attenuation value $\eta$, we estimate $\sigma_u^2 = 4P_s|\eta|^2|h_{tr}|^2 \sum_{l=0}^{N_{st}-1} |h_{st}(l)|^2$.

In our manuscript, the time duration of each backscatter transmitter symbol is set to one OFDM symbol period. The designed waveform $x(n)$ has similar characteristic with frequency modulation, thus it is easily implemented for low-powered devices. Moreover, with the use of CP at the beginning of the OFDM signals and guard intervals, we can eliminate the multipath distortion.

**Comment 2.6.** Here are a few grammatical issues I noticed:

1. On page 1, “Therefore, the AmBC is the key building block that enables internet-of-things and ubiquitous communication among devices with cheap and nearly zero maintenance.” You may want something more like, “Therefore, AmBC is the key building block that enables internet-of-things and ubiquitous communication for cheap devices that require nearly zero maintenance.”

2. On page 4, “We begin by exterminating the following detection problem . . . ” instead of exterminating you probably meant examining.
3. On page 6, “They also showed that the their design was comparable . . . ” delete the extra the between that and their.

4. On page 7, “. . . while keeping the Gaussian approximations to be valid” delete “to be”

Here are a few suggested additional references:


Reply 2.6. We appreciate the reviewer’s careful and detailed corrections on the grammatical issues. We have carefully revised all the sections of the manuscript including those parts commented by the reviewer. Many paragraphs were modified or added to improve readability and clarity, and they are marked in blue color in the revised manuscript.

We also would like to thank the reviewer for pointing out additional references. We have carefully reviewed and cited those relevant suggested references which are [1], [2], [7], [14]-[16], and [19] in the revised manuscript. For the purpose of convenience, we give below the number of references mentioned in the reviewer’s comment.

Signal Detection for Ambient Backscatter Communication with OFDM Carriers

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Abstract: Ambient backscatter communication (AmBC) is considered as a promising future emerging technology. Several works on AmBC have been proposed thanks to its convenience and low cost property. This paper focuses on finding the optimal energy detector at the receiver side and estimating the corresponding bit error rate for the communication system utilizing the AmBC. Through theoretical and numerical analyses, we present two important results. First, we improve the existing energy detector by calculating the optimal averaging power orders. Second, we take the advantages of the early work on orthogonal frequency division multiplexing (OFDM), where the repeating structure of ambient OFDM signals is exploited to cancel out the direct-link interference by using cyclic prefix, then provide a test statistic in which optimal detection threshold and optimal power order are derived accordingly. The study of this paper reveals the inherent limitation of AmBC energy detector and provides a guidance for achieving the optimal power order for a given significance level.

Keywords: ambient backscatter communication; energy detector; OFDM; test statistic

1. Introduction

Ambient backscatter communication (AmBC) is a new mechanism in which a device can communicate with others by backscattering ambient radio-frequency (RF) signals (e.g., WiFi, TV signals) without any additional power suppliers [1,2]. In traditional backscatter communication such as radio-frequency identification (RFID system), a device conveys the data by modulating its reflections of an incident RF signal, which takes an expensive process for generating radio waves. For instance, a reader generates a continuous carrier wave then broadcasts it. A tag receives the signal and modulates it, and backscatters to the reader. Thus, the backscattered signal has a long delay and additional path loss. Moreover, as communicating and computing devices become smaller and abundant, powering them becomes more difficult because they require more batteries, cost, and recharging/replacement that is impractical at large scales. The AmBC solves this problem by utilizing existing RF signals, rather than generating their own radio waves. Since the RF signals are reused, the AmBC is more power-efficient and much cheaper than the traditional radio communication [3–5]. Therefore, the AmBC is the key building block that enables internet-of-things (IoT) and ubiquitous communication among devices with cheap and nearly zero maintenance.

Basically, RF-power devices employing the AmBC must face three main challenging issues [5,6]. First, since the backscatter signals are weak, the problem of signal detection with small changes needs
to be investigated. Second, traditional backscatter receivers are constructed from active components (e.g., oscillator), while the AmBC ones are battery-free [7]. By taking the advantage of existing RF signals in the air, it does not require any additional deployment like the RFID reader that suffers more installation and maintenance costs. The question is either how to build a network that enables ambient backscattering or build new complex digital signal processing techniques. Third, how to operate a distributed multiple access protocol and supporting the functionalities required for the AmBC should be considered. In this paper, we attempt to solve the first problem, which is the design of reader detector to recover the tag bits. There are several related works on this topic such as [5,8,9]. In [5], the authors first performed energy detection without the ability to directly measure the energy on the medium. The key insight is that if the transmitter backscatters information at a lower rate than the ambient signals, then one is able to design a receiver that can separate the two signals by leveraging the difference in communication rates. Thus, the results are very low signal-to-noise ratio (SNR) decoding and low data rate. In [8,9], the authors focused on the uplink signal detection of the communication systems adopting the AmBC, where the detectors exploit maximum \textit{a posteriori} probability (MAP) and maximum-likelihood (ML) estimators at the receiver side. However, the solutions do not perform well when the difference between the backscatter channel and the direct-link channel is small. Other approaches [10,11] make use of WiFi backscatter to decode the tag bits by detecting the changes in the received signal strength which highly depend on channel and multi-path effects. Recently, a new AmBC over orthogonal frequency division multiplexing (OFDM) signals was proposed in [12], where the system model for such AmBC system from spread-spectrum perspective was established. By inhibiting the effects of cyclic prefix (CP) on the ambient OFDM signals, the authors developed a test statistic that is able to invalidate the inter-symbol interference among them. An extension of [12] to the case of multi-antenna receiver was presented in [13], where the test statistic was built from a linear combination of the per-antenna test statistics.

Inspired by [12], our approach is to focus on the uplink signal detection and the performance analysis for a communication system that utilizes AmBC over ambient OFDM signals. Our main ideas and contributions are highlighted as follows. First, we introduce the system model for the AmBC over ambient OFDM carriers in the air and the test statistic for tag signal detection, which are established in [12]. Second, we design an improved energy detector by proposing an arbitrary positive power operation on the signal amplitude instead of the squaring operation given as the previous work. Numerical results demonstrate that the proposed detector with optimum power order can achieve lower bit error rate (BER) and higher data rate than those in [12].

The remainder of this paper is organized as follows. Section II presents the system model and the problem formulation. Section III analyzes the optimal energy detector design for the proposed scheme. Section IV gives the numerical results, followed by the conclusion in Section V.

2. System Model and Problem Formulation

The aim of this section is first to introduce the existing AmBC system that utilizes OFDM carriers, then to present those factors that affect the detector performance. In order to understand how to make use of both energy harvesting and backscattering, we investigate the AmBC system model where an energy harvesting tag uses the existing radio signals from ambient sources to operate itself (e.g., RF source). It produces a modulated reflection of those signals to a nearby receiver (e.g., reader). Legacy receivers employ OFDM structure, where data is transmitted in parallel on a certain number of sub-carriers of different frequencies. Thus, it leads to low power consumption on data transmissions which is very suitable for low-powered hardware or batteryless IoT devices. Moreover, in terms of efficient spectrum sensing, it has been shown that energy detection approach has low computational complexity and ability to identify the spectrum holes without \textit{a priori} knowledge of primary characteristic [14–16]. Once we develop an appropriate test statistic for energy detector, it can guarantee a desired detection performance.
2.1. Notations

The following notations are used throughout the paper.

- $E(\cdot)$ and $\text{Var}(\cdot)$ denote the expectation and variance operators, respectively.
- $\mathcal{N}(\mu, \sigma^2)$ and $\mathcal{CN}(\mu, \sigma^2)$ denote the Gaussian and the circularly-symmetric Gaussian distributions with mean $\mu$ and variance $\sigma^2$, respectively.
- $\text{Re}\{\cdot\}$ is the real part of a complex number.
- A random variable $X$ that is gamma-distributed with shape $k$ and rate $\theta$ is denoted as $X \sim \Gamma(k, \theta)$.

The corresponding probability density function (PDF) in the shape-rate parametrization is

$$f(x; k, \theta) = \frac{1}{\theta^k \Gamma(k)} x^{k-1} \exp\left\{-\frac{x}{\theta}\right\},$$

where $\Gamma(k) = \int_0^\infty x^{k-1} e^{-x} dx, k \in (0, \infty)$ is the gamma function evaluated at $k$.

2.2. Overall System Architecture

The overall system architecture utilizing the AmBC over OFDM carriers is illustrated in Fig. 1. In this system, we consider two communication components coexist: the legacy OFDM system and the AmBC system. In the legacy OFDM system, the RF source transmits OFDM signals to its legacy users, while in the AmBC system a backscatter tag transmits its modulated signals to the reader over ambient OFDM carriers from the RF source. Note that this tag is equipped with a switch that can split the received signal into two parts: information decoder (ID) and energy harvester (EH). Assume that they are all connected to a single antenna and use the same RF signals. The RF-powered passive tag communicates with the reader by switching its antenna impedance of its backscattered signals. The energy harvester collects the energy from the ambient OFDM signals and uses it to provide a small amount power required for the communication and performing tasks at the tag. Finally, the backscattered signal is received and decoded by the reader.

Mathematically, the RF source transmit a passband signal $\hat{s}(n) = \text{Re}\left\{\sqrt{P_s} s(n) \exp\{j\omega n f_c r/2\}\right\}$, where $s(n)$ is the equivalent complex baseband signal with unit power, $P_s$ is the average transmit power, $f_c$ is the carrier frequency, and $f_s$ is the OFDM bandwidth. The tag receives the RF source signal and transmit its modulation signal $\hat{c}(n)$ to the reader. When we add the CP, the ambient OFDM signals are converted to a serial form and transmitted through a wireless channel. Suppose that the channel impulse response of a multipath channel is modeled as a finite impulse response filter with a certain number of taps. We denote $N_{st}$, $N_{tr}$, and $N_{sr}$ as the number of taps corresponding to $h_{st}(n)$, $h_{tr}(n)$, and $h_{sr}(n)$, respectively. Here, we define the maximum delay of the multipath channels as

![Figure 1](image-url)

**Figure 1.** A communication system utilizing the AmBC over OFDM carriers. The AmBC system consists of three main components: RF source (e.g., TV tower), ambient backscatter transmitter (e.g., AmBC tag), and ambient backscattering receiver (e.g., reader), while the legacy OFDM system consists of several legacy receivers (e.g., mobile phones).
We define the detection SNR as $\gamma$, where the square function partitions the observation domain $R$ into two disjoint sets $R_0$ and $R_1$, where

\[ R_0 = \{ x : \delta(x) = 0 \}, \quad R_1 = \{ x : \delta(x) = 1 \}. \]
We also observe that we have two possible incorrect decisions: (i) probability of false alarm, $P_f$ (type-I error) and (ii) probability of miss detection, $P_m(\delta) = 1 - P_d(\delta)$ (type-II error), where $P_d(\delta)$ is the probability of correct detection. Mathematically, we express

$$P_f = P(H_1 \text{ was chosen when } H_0 \text{ true}),$$

$$P_d = P(H_1 \text{ was chosen when } H_1 \text{ true}).$$

In [17], Neyman and Pearson formulated the binary hypothesis testing problem pragmatically by selecting the test $\delta$ that maximizes $P_d(\delta)$ or equivalently that minimizes $P_m(\delta)$, while ensuring that $P_f(\delta)$ is less than or equal to a number $a$. The energy detector is derived by using the generated likelihood ratio test approach [17], where $u(n) \sim CN(0, \sigma_u^2)$ and $v(n) \sim CN(0, \sigma_v^2)$.

$$L(x) = \frac{f_1(z)}{f_0(z)} \frac{H_1}{\partial \theta_0} \geq \tau, \quad (7)$$

where $\tau$ is chosen such that $P_f = \int_{L(z) > \tau} f(z|H_0)dz = \alpha$. We define $z = \{z(n)\}, u = \{u(n)\}, v = \{v(n)\} \ (n = L - 1, \cdots, N_{cp} - 1),$ and $D = N_{cp} - L + 1$. For our hypotheses $H_0$ and $H_1$, the PDFs of the samples can be derived as

$$f_0(z) = \frac{1}{(2\pi\sigma_u^2)^{D/2}} \exp \left\{ -\frac{\sum_{n=L-1}^{N_{cp}-1} |z(n)|^2}{2\sigma_u^2} \right\}, \quad (8)$$

$$f_1(z) = \frac{1}{(2\pi\sigma_v^2)^{D/2}} \exp \left\{ -\frac{\sum_{n=L-1}^{N_{cp}-1} |z(n) - u(n)|^2}{2\sigma_v^2} \right\}. \quad (9)$$

Considering the same detection problem of (5), we define a new test as the following to improve the detection performance.

$$t \overset{\Delta}{=} \frac{1}{D} \sum_{n=L-1}^{N_{cp}-1} \frac{|z(n)|^p}{\sigma_v^p} \frac{H_1}{\partial \theta_0} \geq \tau. \quad (10)$$

Here, $p > 0$ is an arbitrary constant which is discussed later, and $\tau$ is the detection threshold to be determined. Then, the test statistic follows the Gamma distribution with shape $k_i$ and scale $\theta_i$, i.e., $t \sim \Gamma(k_i, \theta_i)$ under $H_i$, where $k_i = \frac{E(D|H_i)}{\text{Var}(D|H_i)}, \theta_i = \frac{\text{Var}(D|H_i)}{E(D|H_i)}$ ($i = 0$ or 1). We denote $F_0(\cdot), F_1(\cdot)$ are the cumulative distribution functions (CDFs) of the Gamma variable $t$ under $H_0$ and $H_1$, respectively, thus $F_i(z;k_i;\theta_i) = \int_0^z \frac{1}{\theta_i^k \Gamma(k_i)} x^{k-1}e^{-x/\theta_i}dx \ (i = 0, 1)$. Then, we have

$$P_f = P(t > \tau|H_0) = 1 - F_0(\tau;k_0,\theta_0), \quad (11)$$

$$P_d = P(t > \tau|H_1) = 1 - F_1(\tau;k_1,\theta_1). \quad (12)$$

To set the threshold, we set $P_f = a$ and thus $\tau = F_0^{-1}(1 - a, k_0, \theta_0)$, resulting in $P_d = 1 - F_1(F_0^{-1}(1 - a, k_0, \theta_0); k_1; \theta_1)$. According to the central limit theorem (CLT) [17], as $D$ becomes large we can represent $t \sim \mathcal{N}(E(t), \text{Var}(t))$. By assuming that $|z(n)|^p/\sigma_v^p$ are independent and identically distributed random variables, we obtain

$$E(t|H_0) = \mu_0; \quad \text{Var}(t|H_0) = \frac{\sigma_0^2}{D}, \quad (13)$$

$$E(t|H_1) = \mu_1; \quad \text{Var}(t|H_1) = \frac{\sigma_1^2}{D}. \quad (14)$$
Here, we have

\[
\begin{align*}
\mu_0 &= \frac{2^{p/2}}{\sqrt{\pi}} \Gamma \left( \frac{p+1}{2} \right), \\
\mu_1 &= \frac{2^{p/2}(1+\gamma)^{p/2}}{\sqrt{\pi}} \Gamma \left( \frac{p+1}{2} \right), \\
\sigma_0^2 &= \frac{2^p}{\sqrt{\pi}} \left[ \Gamma \left( \frac{2p+1}{2} \right) - \frac{1}{\sqrt{\pi}} \Gamma^2 \left( \frac{p+1}{2} \right) \right], \\
\sigma_1^2 &= \frac{2^p(1+\gamma)^p}{\sqrt{\pi}} \left[ \Gamma \left( \frac{2p+1}{2} \right) - \frac{1}{\sqrt{\pi}} \Gamma^2 \left( \frac{p+1}{2} \right) \right],
\end{align*}
\] (15)

where \( \Gamma(k) = \int_0^\infty x^{k-1}e^{-x}dx \) \( (k > 0) \) is the Gamma function evaluated at \( k \). Thus, the probabilities of false alarm and correct detection can be evaluated as

\[
P_f \approx Q \left( \frac{\tau - \mu_0}{\sigma_0/\sqrt{D}} \right), \quad P_d \approx Q \left( \frac{\tau - \mu_1}{\sigma_1/\sqrt{D}} \right).
\] (18)

To set the threshold, we have \( P_f = \alpha \), and thus \( \tau = Q^{-1}(\alpha)\sigma_0/\sqrt{D} + \mu_0 \), resulting in

\[
P_d \approx Q \left( \frac{Q^{-1}(\alpha)\sigma_0 + \sqrt{D}(\mu_0 - \mu_1)}{\sigma_1} \right).
\] (19)

Thus, the overall BER is given by

\[
P_e = \pi_0 P_f + \pi_1 (1 - P_d).
\] (20)

Considering equal probabilities of each type of error, i.e., \( \pi_0 = \pi_1 = 1/2 \), the minimum value of BER is achieved by taking the derivative of \( P_e \) with respect to \( \tau \) and letting it to zeros, resulting the optimal detection threshold \( \tau^* \). The detailed derivation is given in the Remark 1.

**Remark 1** (Optimal value of detection threshold). In order to find the optimal value \( \tau^* \), we need to solve the following equation

\[
\frac{1}{\sqrt{2\pi\sigma_0^2/D}} \exp \left\{ -\frac{(t - \mu_0)^2}{2\sigma_0^2/D} \right\} = \frac{1}{\sqrt{2\pi\sigma_1^2/D}} \exp \left\{ -\frac{(t - \mu_1)^2}{2\sigma_1^2/D} \right\}.
\] (21)

By taking the natural logarithm on both sides, (21) can be simplified as

\[
\frac{D}{2} \left[ \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2} \right] t^2 + D \left[ \frac{\mu_0}{\sigma_0^2} - \frac{\mu_1}{\sigma_1^2} \right] t + \left[ \ln \frac{\sigma_0}{\sigma_1} + \frac{D}{2} \left( \frac{\mu_0^2}{\sigma_0^2} - \frac{\mu_1^2}{\sigma_1^2} \right) \right] = 0.
\] (22)

The above equation is a quadratic form, thus the optimal detection threshold is given by

\[
\tau^* = -\frac{\xi_1}{2} + \sqrt{\left(\frac{\xi_1}{2}\right)^2 - \xi_2},
\]

where

\[
\frac{\xi_1}{2} = \left( \frac{\mu_0}{\sigma_0^2} - \frac{\mu_1}{\sigma_1^2} \right) \left( \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2} \right), \quad \xi_2 = \frac{\xi_1^2}{4} \ln \frac{\sigma_0}{\sigma_1} + \frac{\mu_0^2}{\sigma_0^2} - \frac{\mu_1^2}{\sigma_1^2}.
\] (23)
**Remark 2** (Optimal value of power order $p$). In [12], the value $p$ is fixed at $p = 2$. In our paper, the value $p$ is chosen to maximize $P_d$ at fixed $P_f$, $\gamma$, and $D$. Thus, the optimal value of $p^*$ is obtained by solving

$$p^* = \arg \max_p P_d = \arg \max_p Q \left( \frac{Q^{-1}(\alpha) \sigma_0 / \sqrt{D} + \mu_0 - \mu_1}{\sigma_1 / \sqrt{D}} \right).$$

If we assume $P_d$ to be differentiable, then we can differentiate both sides

$$\frac{\partial P_d}{\partial p} = 0.$$

The derivation of solving this equation is given in Appendix A.

4. Numerical Results

In [12], the authors compared the energy detector with a benchmark design in which the reader detected the tag bit by distinguishing between two different orders of the average power of the received signal $y(n)$, where

$$\bar{z} \triangleq \frac{1}{N + C_{cp}} \sum_{n=1}^{N+N_{cp}-1} |y(n)|^2.$$

They also showed that their design was comparable to the benchmark design in terms of complexity, but the performance was better in terms of transmission rate and BER. Thus, we compare our proposed approach with [12] (i.e., $p = 2$), which is referred as the “conventional” energy detector. A summary description of simulation parameters is given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM bandwidth</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Number of paths</td>
<td>$N_{tr} = N_{st} = 8$, $N_{tr} = 1$</td>
</tr>
<tr>
<td>Attenuation value</td>
<td>$\eta = 1$</td>
</tr>
<tr>
<td>CP length $N_{cp}$</td>
<td>16, 32, 64, 128</td>
</tr>
<tr>
<td>Number of carriers $N$</td>
<td>$8N_{cp}$</td>
</tr>
</tbody>
</table>

In the following, we briefly describe some metrics that we used to evaluate the proposed statistic test. The test must be a sufficient statistic for our energy detection problem and contains all the information required to distinguish two hypotheses $H_0$ and $H_1$.

(i) First, we check the validity of the Gaussian approximation for the proposed test. In fact, since the test statistic (10) follows the Gamma distribution under both hypotheses, the length $D$ must be large enough to apply the CLT while not very large to keep the approximation meaningful. In the first run, Fig. 2 gives us a case study on this approximation.

(ii) Second, in order to find the optimal $p$ for the test (10) instead of using $p = 2$, we solve (24) to obtain an adaptive power order. The result is shown in Fig. 3. The purpose of this result is to observe how the $p^*$ changes for maximizing the probability of correct detection according to the changes in the SNR. Thus, we may have a certain strategy to select $p$ for a given SNR and a false alarm rate.

(iii) Third, with different settings (e.g., SNR and $N_{cp}$), we observe how much the BER changes when using our test statistics and the conventional ones in terms of our ability to solve $p^*$ with high correct detection probability $P_d$. The results are given in Figs. 4 and 5.

(iv) Finally, we provide the median receiver operating characteristics (ROC) curve for our detector design as predicted by our aforementioned analysis.
Following the above construction, in order to verify the accuracy of approximating the simulated PDFs (i.e., Gamma distribution) by the theoretical approximation (i.e., $Q$-function), Fig. 2 illustrates those PDFs when $\gamma = 0$ dB, $N_c = 64$, and $N = 512$, thus, $L = 8$ and $D = 57$. We observe that the Gamma approximation fits well in most cases considered. The accuracy of the approximation increases when $p$ decreases. This approximation may be adequate for practical energy detectors since we can improve it by increasing the length $D$ in the detection. Thus, we have to select appropriate values of length $D$ and $P_f$ to guarantee high probability of detection, while keeping the Gaussian approximations to be valid.

**Figure 2.** Illustration of CDFs under $H_0$ and $H_1$. Note that the theoretical analysis shows that the test statistic $t \sim \Gamma(k_i, \theta_i)$ under $H_i$, while the simulation approximation gives us $t \sim \mathcal{N}(E(t), \text{Var}(t))$, where $E(t)$ and $\text{Var}(t)$ are given in (13) and (14), respectively.

Fig. 3 shows the optimum value of $p$ in (24) with different fixed values of $P_f = \alpha$. The value $p^*$ maximizing $P_d$ decreases when $\gamma$ increases. We also plot a small subgraph at the right hand side
of Fig. 3 to illustrate the $p^*$ value (in vertical axis) versus small SNR (in horizontal axis) because we observe that $p^*$-curve has a big jump in its value for $\gamma$ in the range of $(0, 1.5)$. We offer some brief comments. First, the value $p^*$ is probably the best solution we has achieved through numerical results. The $p^*$-curve grows sub-linearly versus small value of SNR (e.g., $\gamma \leq 1$) simultaneously, while it decays slowly and remains constant as a function of $\gamma \geq 6$. In a certain sense, the vector $z$ represents the relative difference between two input signals. Due to weak backscattered signals, if it has any small component, $p = 2$ makes them negligible. On the other hand, the optimal power order is around 1, which is more irritated by small values. For instance, when $\gamma \leq 1$, the proposed detector returns $p^* \approx 1$ rather than $p = 2$. In this case, it prefers returning the number of non-zero values of the vector $Z = \{z(n)\}$ rather than tolerates them.

![Figure 3](image)

**Figure 3.** The optimum value of power order $p$ versus $\gamma$. It shows the effect of $\gamma$ on $p^*$ at several different SNR levels, which comes from solving procedure of (25).

Fig. 4 demonstrates the theoretical BER performance of the conventional energy detector with $p = 2$ and the proposed detector with optimal power order $p^*$. We set $N_{cp} = 64$, $N = 512$, and $\alpha = 0.01$. We can see that the energy detector performs worse than the proposed one because of the inaccurate Gaussian approximation. We also see that the improvement here becomes more prominent when the target probability of false alarm is smaller, as well as the achieved SNR is relatively higher. The BER becomes flat even for high SNR value. For the proposed detector, it can be found that the increasing SNR yields reduced BER, especially when SNR is small (e.g., SNR $\leq 2$ dB). For larger SNR, the BER performance remains unchanged. This phenomenon may be caused by the strong direct-link interference [13]. Moreover, since the reader tries to distinguish between two bits by taking a sufficiently large number of samples, i.e., $D$, the value of $D$ we consider here is large enough to apply the CLT, thus the probability of miss detection and the probability of false alarm are moderate, i.e., they are not changed with $D$. Consequently, the overall BER in (20) does not become much different. The detector must reach the error probabilities uniformly over a whole uncertainty set with various $D$. As SNR increases, it hits the SNR wall [19] while the required sample complexity meets our performance target.

Fig. 5 depicts the curves of BER versus SNR with several value of $N_{cp}$ for the proposed detector. We set $\gamma = 5$ dB and $\alpha = 0.01$. The BER approaches 0.5 at small $N_{cp}$, and there exist little gaps between
BER curves. We observe that the BER decreases as \( N_{cp} \) increases. However, the smaller \( N_{cp} \) offers higher data rate from the relationship

\[
R_{tag} = \frac{f_s}{(N + N_{cp})},
\]

(27)

where \( R_{tag} \) is the tag rate [12]. Obviously, if we fix the number of OFDM carriers \( N \), \( R_{tag} \) decreases as the CP length increases, while the BER decreases, as illustrated in Fig. 5. Thus, there exists a trade-off between the BER and the data rate \( R_{tag} \).

Figure 4. BER versus SNR \( \gamma \). We observe that BER achieves the maximum at \( \gamma = 0 \). The designed detector can perform well even when the SNR is high.

Figure 5. BER versus SNR with \( N_{cp} \). As we predicted, the BER increases as \( N_{cp} \) decreases.
small, i.e., the performance gain becomes much larger in the lower SNR environment. As the SNR increases, the detection threshold must be set higher to obtain a good ROC.

![ROC curve with SNR γ](image)

Figure 6. ROC curve with SNR \( \gamma \). An ROC curve is obtained by taking the average over 100 independent trials.

5. Conclusion

In this paper, we have studied the signal detection for the AmBC system with OFDM carriers, while providing some key mathematical insights underlying this theory and proposing an improved energy detector with optimum power orders. Especially, in order to maximize the probability of correct detection, the power order of energy detector was chosen subject to the target probability of false alarm. The proposed detector was shown to improve energy efficiency for spectrum sharing via AmBC.

Moreover, based on the insightful results we suggest the following directions for future work.

(i) Regarding tag operation, an important direction is to come up with a model that examines the energy harvesting model and enhances the detection performance accordingly.

(ii) In our problem formulation, we use a simple noise uncertainty model, i.e., the variance of \( v(n) \) is assumed to be bounded by a given number \( B \). This value depends only on a single value \( \rho \), thus it may not incorporate the RF strength and other changes in the environments. Therefore, we need to investigate other nonlinear models that relate to energy detector’s inherent noise uncertainty.

(iii) We have shown that the proposed energy detector can be effective for the AmBC system with OFDM carriers. However, it needs more theoretical bounds on \( D \) and SNR \( \gamma \) along with numerical results.

(iii) In the problem formulation, we assumed that the tag has two states: backscattering and non-backscattering, while in practice its antenna load may switch among three states: no reflecting, reflecting in the same phase, and reflecting in the opposite phase, resulting in a ternary signal \( B(n) \). Thus, we need to design a waveform \( x(n) \) to convey the corresponding bits.

We expect that the above future directions can contribute to the advancement of energy detection and estimation areas.

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the work, provided the funding, supervised the research and reviewed the draft of the paper. All authors discussed
the results, approved the final version, and wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

- AmBC Ambient backscatter communication
- BER Bit error rate
- CDF Cumulative distribution function
- CLT Central limit theorem
- CP Cyclic prefix
- ED Energy harvester
- ID Information decoder
- MAP Maximum a posteriori probability
- ML Maximum likelihood
- OFDM Orthogonal frequency-division multiplexing
- PDF Probability density function
- RF Radio frequency
- RFID Radio-frequency identification
- ROC Receiver operating characteristic
- SNR signal-to-noise ratio
- TV Television

Appendix A. Solving (25)

As we mentioned before, the optimal value $p^*$ can be obtained by simply taking the derivative of $P_d$ and setting it to be zero. The detailed procedure is described as follows. By using the chain rule and the fundamental theorem of calculus to find the derivative of $\frac{\partial P_d}{\partial \Lambda}$, we rewrite (25) as

$$\frac{\partial P_d}{\partial \Lambda} \frac{\partial \Lambda}{\partial p} = 0, \quad (A1)$$

where $\Lambda = \left[ Q^{-1}(a) \sigma_0 + \sqrt{D}(\mu_0 - \mu_1) \right] / \sigma_1 \triangleq A / \sigma_1$. With some mathematical manipulations, it can be shown that

$$\frac{\partial P_d}{\partial \Lambda} = -\frac{1}{2\pi} e^{-\frac{\Lambda^2}{2}}, \quad \frac{\partial \Lambda}{\partial p} = \frac{\partial A}{\partial p} \sigma_1 - A \frac{\partial \sigma_1}{\partial p}. \quad (A2)$$

Defining $\Psi(x) = \frac{d}{dx} \ln \Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ which is known as the $\Psi$-function [18], we have

$$\frac{\partial A}{\partial p} = Q^{-1}(a) \frac{d \sigma_0}{dp} + \sqrt{D} \left( \frac{d \mu_0}{dp} - d \mu_1}{dp} \right), \quad (A3)$$

$$\frac{d \mu_0}{dp} = \frac{2^{p/2-1}}{\sqrt{\pi}}\Gamma \left( p + \frac{1}{2} \right) \ln 2 + \Psi \left( p + \frac{1}{2} \right), \quad (A4)$$

$$\frac{d \mu_1}{dp} = \frac{2^{p/2-1}(1+\gamma)^{p/2}}{\sqrt{\pi}} \Gamma \left( p + \frac{1}{2} \right) \times \left[ \Psi \left( p + \frac{1}{2} \right) + \ln(2 + 2\gamma) \right], \quad (A6)$$

$$\frac{d \sigma_1}{dp} = \frac{2^{p/2-1}(1+\gamma)^{p/2}}{\sqrt{\pi}} \left[ B^2 \ln(2 + 2\gamma) + \Psi \left( p + \frac{1}{2} \right) \right] \Gamma \left( p + \frac{1}{2} \right)$$

$$\times \Gamma \left( p + \frac{1}{2} \right) - \frac{1}{\sqrt{\pi}} \Psi \left( p + \frac{1}{2} \right) \Gamma^2 \left( p + \frac{1}{2} \right), \quad (A7)$$
where $B = \sqrt{\Gamma \left( \frac{p + 1}{2} \right) - \frac{1}{\sqrt{\pi}} \Gamma \left( \frac{p + 1}{2} \right)}$. By substituting $(A3)-(A7)$ into $(A2)$, the solution of (25) can be numerically found. With other fixed parameters, $P_d$ is a function of the single variable $p$. Thus, we first select an guess interval for $p$, then apply efficient numerical tools (e.g., Newton method [17]) to obtain the approximate roots of (A2). We also assume that the error produced due to computing process can be ignored, i.e., the numerical result is acceptable for all cases.

References