Referee's Comments

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Manuscript Title: Degree-constrained minimum spanning hierarchies in graphs
Author(s): Miklos Molnar

The manuscript, titled "Degree-Constrained Minimum Spanning Hierarchies in Graphs," addresses the significant problem of finding degree-constrained minimum spanning hierarchies (DCMSH) in graphs, with a particular focus on scenarios where the degree constraints represent momentary capacities rather than fixed budgets. This problem is highly relevant, especially in network optimization contexts like optical networks. The manuscript makes a notable contribution by introducing spanning hierarchies as a generalization of spanning trees, offering exact and heuristic solutions, and analyzing approximation possibilities. The technical rigor is evident in the precise definitions, logical structuring of lemmas and theorems, and the application of integer linear programming (ILP) for exact solutions. Furthermore, the comprehensive analysis of the existence conditions for DCMSH, paired with a detailed comparison of results across random graphs, underscores the manuscript's strength in providing practical insights into the performance of the proposed algorithms. However, minor revisions are recommended to enhance the paper's clarity and accessibility further, as detailed below.

1. Page 1, Line 1: "Minimum spanning tree problem ..."
   Revision: "**The** minimum spanning tree problem ..."

2. Page 1, Line 2: "... do not always exist and the optimum ..."
   Revision: "... do not always exist**,** and the optimum ..."

3. Page 1, Line 4: "(For a given node, ...)"
   Revision: "For a given node, ..."

4. Page 1, Line 8: "We study the conditions of its existence, we propose ..."
   Revision: "We study the conditions of its existence, **and** we propose ....."

5. Page 1, Line 14: "... for instance for efficient broadcast ..."
   Revision: "... for instance**,** for efficient broadcast ..."

6. Page 1, Line 22: "... a graph respecting *budge* type degree bounds ..."
   Revision: "... a graph **adhering to** *budge*-type degree bounds ..." This version uses "adhering to" as an alternative to"respecting," which may sound more formal. It is recommended to replace the use of "respect" throughout the manuscript with a similar phrasing.

7. Page 1, Line 25: "... hard to solve and unfortunately, ..."
   Revision: "... hard to solve**,** and unfortunately, ..."

8. Page 1, Line 34: "... are possible and the spanning tree structure ..."
   Revision: "... are possible**,** and the spanning tree structure ..."

9. Page 2, Line 42: " How can **be** the optimum calculated?"
   Revision: "How can the optimum **be** calculated?"

10. Page 2, Line 49: "Necessary and sufficient conditions ..."
    Revision: "**The necessary** and sufficient conditions ..."

11. Page 2, Line 70: "... $H = (U, D)$ where ..."
    Revision: "... $H = (U, D)$**,** where ..."

12. Page 2, Line 71: "... concered nodes and D is ..."
    Revision: "... concered nodes**,** and D is ..."

13. Page 3, Line 95: "... a spanning tree can exist iff ..."
    Revision: "... a spanning tree can exist **if and only if** ..."

14. Page 3, Line 99: "... constraint**,** and the second to..."
    Revision: Remove the comma.

15. Page 4, Line 118: "... the node set V, s.t. the degree ..."
    Revision: "... the node set V **such that** the degree ..."

16. Page 4, Line 120: "Minimum cost structure ..."
    Revision: "**The minimum** cost structure ..."

17. Page 4, Line 126: "... constraints which is not ..."
    Revision: "... constraints**,** which is not ..."

18. Page 4, Line 131: "... is unitary and the capacity constraints are ..."
    Revision: "... is unitary**,** and the capacity constraints are ..."

19. Page 4, Line 132: "... degree constrained spanning tree ..."
    Revision: "... **a** degree**-**constrained spanning tree ..."

20. Page 5, Line 160: "... there are no possible paths ..."
    Revision: "... there are no paths ..."

21. Page 6, Line 204: "... s.t. ..."
    Revision: "... such that ..."

22. Page 6, Line 208: "... s.t. ..."
    Revision: "... such that ..."

23. Page 8, Line 279: "Equals to the number ..."
    Revision: "It is equals to the number ..."

24. Page 8, Definition of $F(m, n)$: "Denotes the quantity ..."
    Revision: "It denotes the quantity ...".

25. On page 8, please add periods at the end of Equations (1), (2), (3), (4), and (5), respectively.

26. Page 8, Line 288: "... iff ..."
    Revision: "... if and only if ..."

27. Page 8, Line 291: "... non source node ..."
Revision: "... non-source node ..."

28. Page 8, Line 296: "... non source node ..."
Revision: "... non-source node ..."

29. On page 9, please add the periods at the end of Equations (6), (7), (8), (10), and (11), respectively.

30. Page 9, Line 298: "... the full coverage."
Revision: "... the full coverage as follows."

31. Page 9, Line 301: "... nodes.)"
Revision: "... nodes.) We have".

32. Page 9, Line 302: "... only one flow."
Revision: "... only one flow. We require"

33. Page 9, Line 30: "Flows transit by occurrences of arcs."
Revision: "The conditions of flow transit by occurrences of arcs are as follows."

34. On page 9, please add ", and" at the end of Equation (9).

35. Page 9, Line 302: "... in [16])."
Revision: "... in [16]). We have".

36. Page 9, Line 317: "... if the number ... are ..."
Revision: "... if the number ... **is** ..."

37. Page 9, Line 318: "... the number of incoming and outgoing arcs respectively."
Revision: "... the **numbers** of incoming and outgoing arcs**,** respectively."

38. Page 9, Line 327: "... s.t. ..."
Revision: "... such that ..."

39. On page 10, in the third line, please replace "By construction" with "By the construction".

40. On page 10, add "We obtain" at the end of the sixth line and a period at the end of the seventh line.

41. Page 10, Line 353: "... s.t. ..."
Revision: "... such that ..."

42. On page 10, please add the periods at the end of lines 357 and 358, respectively.

43. Page 11, Line 389: "... in the tree (otherwise... is unnecessary)."
Revision: "... in the tree; otherwise... is unnecessary."

44. Page 11, Line 399: "... with central node c ..."
Revision: "... with **a** central node c ..."

45. Page 11, Line 407: "... are needed and some edges must ..."
Revision: "... are needed**,** and some edges must ..."

46. Page 11, Line 417: "Let us suppose**,** that ... with central node c ..."
Revision: Let us suppose that ... with **the** central node c ..."

47. Page 12, Line 425: "... adjacent edges and the last occurrence ..."
Revision: "... adjacent edges**,** and the last occurrence ..."

48. Page 12, Line 426: "... $D(c) = 3$, $d_p(c) = 2$."
Revision: "... $D(c) = 3$ and $d_p(c) = 2$."

49. Page 12, Line 434: "If there are a few numbers of nodes in V_1 ..."
Revision: "If the number of nodes in V_1 ..."

50. On page 12, please add a period at the end of the last line.

51. Page 13, Line 456: "... with $|V_1| > 2$ a bounded approximation ratio ..."
Revision: "... with $|V_1| > 2$**,** a bounded approximation ratio ..."

52. Page 13, Line 460: "When ϵ tends to ..."
Revision: "When w tends to ..."

53. On page 14, in Table 1, please replace c(MSTSL), c(Heur), c(DCMSH), and c(DCMST) with $c(MSTSL), c(Heur), c(DCMSH)$, and $c(DCMST)$, respectively.

54. Page 15, Line 506: "... was fixed to 100 ..."
Revision: "... was fixed at 100 ..."

55. On page 15, in Table 2, please replace "dupl", c(MSTSL), c(Heur), c(DCMSH), and c(DCMST) with "$dupl$", $c(MSTSL), c(Heur), c(DCMSH)$, and $c(DCMST)$, respectively.

56. Page 15, Line 521: "... increase quasy linearly ..."
Revision: "... increase quasi-linearly ..."

57. Page 15, Line 522: "... let the lines with 80 and 160 nodes be compared."
Revision: "... let us compare the lines with 80 and 160 nodes ..."

58. Page 15, Line 527: "DCMSHs can built ..."
Revision: "DCMSHs can **be** built ..."

59. Page 15, Line 531: "... not always exist, necessary and sufficient conditions ..."
Revision: "... not always exist, **the** necessary and sufficient conditions ..."