Degree-constrained minimum spanning hierarchies in graphs

proposed for **Algorithms /MDPI** by Miklos Molnar

Response to Reviewer 1 Comments

1. Summary

Thank you very much for taking the time to review this manuscript. Please find the detailed responses below and the corresponding revisions/corrections highlighted in red in the re-submitted proposition.

2. Questions for General Evaluation	**Reviewer's Evaluation**	**Response and Revisions**
Does the introduction provide sufficient background and include all relevant references?	Yes	
Are all the cited references relevant to the research?		
Is the research design appropriate?	Yes	
Are the methods adequately described?	Must be improved	Following the comments and indications of the reviewer, the descriptions are modified
Are the results clearly presented?	Must be improved	The presentation is revised
Are the conclusions supported by the results?	Must be improved	The conclusion resumes the results

3. Point-by-point response to Comments and Suggestions for Authors

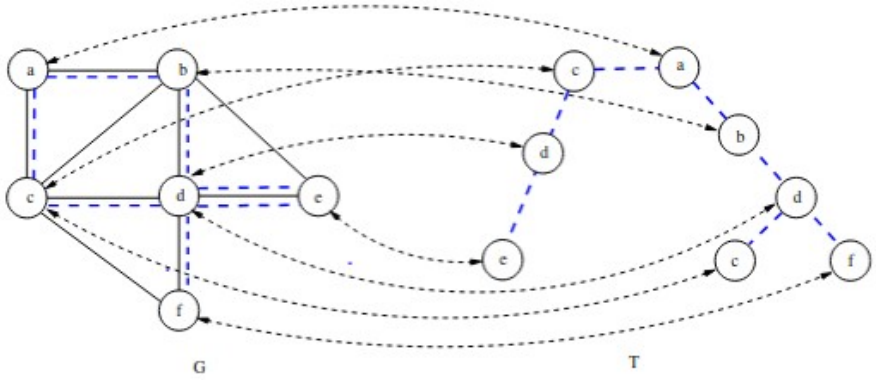
Comments 1: ***The standard of much of the explanation of the paper is not of the standard I would expect in a journal paper. The problem is not in the English (I have only a few minor points), but in the way concepts are dealt with. I imagine that those who have worked on closely related problems would understand most of the paper, but the average reader of Algorithms would struggle. I have no reason to think that anything major in the paper is incorrect.***

Response 1: Thank you for this important comment. I understand the difficulties to read the paper. I had also the same difficulties to formulate the messages, because the domain (homomorphism based graph related structures) are weakly represented in the literature. To present the results on spanning hierarchies, I tried to use different concepts. In the revised version, I simplified the concepts and the presentation. I don't speak on "visit" of nodes

having instantaneous capacities, etc. Hierarchies are presented having eventually multiple occurrences of nodes and these occurrences must satisfy the capacity constraint. I tried to filter confusing useless concepts.

Comments 2: ***In Section 2 it would be helpful in Figure 1 to add arrows to the arcs indicating the mapping, and to expand the caption to explain more clearly what is being shown, and what the various solid dots and dashes are indicating. The figure is really helpful to illustrate the concept of a hierarchy.***

Response 2: Thank you for this advise. I modified the figure and indicated the meaning of the lines (red characters in the text). There is the new figure.



… Figure 1 illustrates with discontinuous lines a hierarchy in a graph and the associated tree. The labels of the nodes in T are the labels of the corresponding nodes in G. Dotted lines with arrows indicate the associations between these nodes.

Comments 3: ***In Section 3 the Definition 3 is a little vague (instantaneous, respected for each visit) and could be presented more formally. Again, Figure 2 is helpful to illustrate this, but the caption should be expanded to explain what is being shown and what the numbers represent.***

Response 3: Thank you for this comment. The modified definition follows.
Definition 3 (Node capacity constrained minimum spanning problems). In a graph G, the degree bounds D(v) associated with nodes v ∈ V represent the maximum degree capacities. The problem consists of finding the minimum cost solution spanning the node set V , s.t. the degree constraints are respected for each occurrence of the nodes.

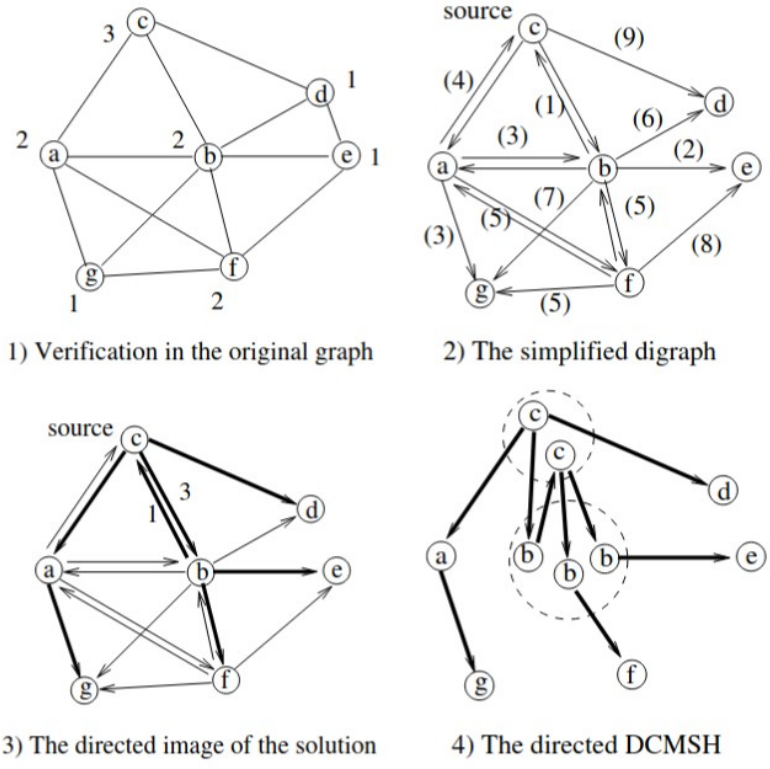
The figure shows a graph G and the labeled tree of its DCMSH. The following explanations are in the text before the figure.
A degree-constrained minimum spanning hierarchy (DCMSH) is illustrated in Figure 2. The cost of any edge in G is unitary and the capacity constraints are indicated. Trivially, DCMST does not exist but there is a spanning hierarchy satisfying the constraints. It contains the node b twice, and each occurrence of this node respects

the degree constraint as shown by the labeled tree T of the solution.

Comments 4: ***In subsection 5.1 the author assumes that the reader will know what an "image" is in this context. Again in Figure 5 the caption needs to be expanded to explain what is being shown, and the various steps in Section 5 should be linked more closely to the illustration in this figure.***
It it is not immediately clear how the four points in 5.1.1 link to Theorem 1 or its preceeding lemmas. Each point should be linked to the appropriate lemma to aid the reader.

Response 4: Thank you for this comment. To facilitate the read of the paper, I added a formal definition of the image concept.
Definition 2 (Image of a hierarchy). *The sub-graph defined by the set of nodes and edges in G associated by h to T is the image of the hierarchy.*

Moreover, Figure 5 is modified, and the steps in Section 5 are indicated explicitly in the figure. And the figures are also explicitly referenced in the text.



As it is indicated in the text, Theorem 1 formulates necessary and sufficient conditions for the existence of degree-constrained spanning hierarchies, and so, it is related only to Step 1). More precisely, the verification in the first step is the application of Theorem 1. The others steps are not concerned by the theorem. The lemmas that precede the theorem are lemmas for the theorem as they are referenced in the proof.

Comments 5: ***Section 5.1.1 is not at all clear. It is not at all clear how the flow is found. What does"one flow to each destination from s" mean exactly? Is Figure 5c meant to be showing a flow? Only two solid arcs have values indicated. Almost all of 5.1.3 and 5.1.4 is rather obscure. I can almost see what is going on from Figure 5, but is is difficult to link it to the rest of the explanation. Almost all of Section 5.1 needs a careful rewriting. Many of the statements made seem rather disjointed and unexplained.***

Response 5: Thank you for this indication. I think, there is a misunderstanding. Section 5.1.1. is the simple application of the result in Theorem 1. The different conditions must be verified (absence of any separators, verification of some sub-sets V_i, i=1, 2,3, …)
The ILP proposed in 5.1.3. is based on a flow computation, as it is the case in [16]. An arbitrary source is selected in the directed graph built in 5.1.2 and a flow is computed from this source to each other node (the whole graph must be covered). More details can be found in the ILP.
The results of the ILP indicate how many times an arc is used in the solution. This information permits the construction of the hierarchy in Step 5.1.4.
The linking of the steps 5.1.1 – 5.1.4 is indicated in the text.

Comments 6: ***lines 344, 347 Should be upper bound not lower bound? (See Table 1), (and all but first two entries in last column of Table 2 ??).***

Response 6: Thank you for this important comment. I agree, the cost of a DCMST can not be lower than the cost of a DCMSH. I deleted this sentence. The interest of this section is to find a lower bound and serve as the base of the approximations.
The modified definition is the following:

5.2.1. A lower bound

Consider the following spanning tree.

Definition 5 (Minimum spanning tree with specified leaves - MSTSL). *Let $V_1 \subset V$ be the subset of desired leaves. The MSTSL is a minimum cost spanning tree covering the node set s.t. the nodes in V_1 are leaves in the tree.*

Comments 7: ***Similar comments can be made about the presentation of the Heuristic and the Approximations***

Response 7: Thank you for this comment. I have revised the remaining sections of the paper. The main modifications are highlighted in red in the new document..

4. Response to Comments on the Quality of English Language

Point 1:Minor points

line 1 Minimum spanning tree problem → The minimum spanning tree problem

...

line 268 Eventual simplifications → Simplifications.

line 270 Be more specific about the meaning of "useless".

line 434 If there are a few numbers of nodes → If there are only a small number of nodes

Response 1: Thank you for the comments. The modifications are in the text.

Several acronyms are used in the paper: (DCMST, DCMSH, APX, MSTSL etc.). It would help the reader if the list of notation were complemented by a list of acronyms

Response 2: Detailed definitions for these acronyms can be found in the paper. Moreover, APX is known for the readers of "Algorithms".

5. Additional clarifications