

Report on the Revised Version of the Paper

Ratio-covarieties of numerical semigroups

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Overview

The referee is pleased to see that authors followed most of his suggestions; the paper looks much better now.

Comments for the new Section 2

This section is somewhat confusing, as the discussion of the main example $\mathcal{R}(F, m)$ is interrupted by Definition 2.2 to Example 2.4, which belong to the next section; this passage should be moved there. On the other hand the section could be enriched by adding a discussion of $\text{MED}(F, m)$ (and maybe also of $\text{ST}(F, m)$, mentioned in the first version of the paper). An example of a ratio-covariety of a different kind (i. e. *not* a subset of $\mathcal{R}(F, m)$) would also be very welcome. The title of the section could then be changed to something like *Basic Properties and Examples*.

Final suggestions

There are still minor changes that should be made; some of them should have been suggested already in the first report:

- Page 2, after Definition 1.2: the most trivial example of a ratio-covariety could be mentioned, the singleton $\{S\}$ with any numerical semigroup S .
- Page 2, Line 64: “denote” typo: *denoted*
- Page 2, Lines 67 and 71: delete the repetition of “In Section 5”
- Page 2, Line 70: “where $\#Y$, denotes” better: delete comma
- Page 2, Line 71: “we will see how...” better: *We will characterize the elements of $\mathcal{R}$ with $\mathcal{R}$ -rank 0 and 1.*
- Page 3, Line 84: the fact “ $\mathbb{N} \setminus T$ is a finite set” should better be mentioned in the proof of Proposition 2.1, for example *As $\mathbb{N} \setminus T$ is a finite set, the set $\{S \mid S \text{ numerical semigroup with } T \subset S\}$ is also finite ...*
- Page 3, Definition 2.2: “We will say that an integer z is a...” better: *An integer z is called a ...*
- Page 4, Definition 3.5: “We will ...” better: *An element $a \in A$ is called maximal with respect to $\leq_S$, if ...*
- Page 5, Lines 155,156: The sentence “Denote ...” should be included into Definition 3.5.
- Page 7, Line 243: “If S ...” better: *For a numerical semigroup S we set*
- Page 8, Line 278: “elementos” Spanish or typo: *elements*
- Page 10, Line 342: missing reference in “Now, by applying Lemma ??, we have that” better: *Since $e(S) \leq m(S)$, this implies*

Conclusion

Granted the improvements mentioned above, the referee is pleased to recommend the paper for publication in *Axioms*.