Manuscript No.: sustainability-2618852

Manuscript Title: *“***Optimal Predictive Torque Distribution Control System to Enhance Stability and Energy Efficiency in Electric Vehicles***”*

Dear Professor Zariah Zou,

Assistant Editor, Sustainability

We would like to express our sincere gratitude to the Associate Editor and the Deputy Editor-in-Chief and the reviewers for your careful reviews of our paper, and your positive recommendations. The editors’ and reviewers’ comments and suggestions have constructively helped us to improve the quality and the presentation of this manuscript further. In light of these comments and suggestions, we have revised the original manuscript duly and carefully. In what follows, we indicate how your comments and suggestions have been taken into account in the revision.

For clarity, we will quote these comments/suggestions in **blue color**. Also, we have highlighted the changes in the paper to clearly address the honorable reviewers' concerns about revision (by special colors of any reviewers).

Best regards

The authors of the manuscript

****************************Associate Editor Comments ****************************

The authors would like to express sincere gratitude to the respected AE for their time and effort dedicated to this paper. The paper is accordingly revised based on the comments.

Comments to the Author:

The comments from the reviewers have to be seriously considered and integrated into the revision. The quality of this study is further improved from the perspectives of motivation, equations, and presentation.

**************** Reviewer 1 (**Red Color in the revised manuscript**) **********************

Dear reviewer, we would like to express our sincere gratitude for your valuable time, and efforts given to review this letter paper. We have comprehensively answered all your comments and carefully addressed them in the paper to cover all your concerns regarding the expectation of revision. Thank you for all your useful and constructive comments, remarks, and questions. We have revised the original manuscript and in response to your comments, have added sections in red into the revised manuscript.

Comments to the Author:

Comment 1: Give the specific meaning of the character RFti in Equation (2);

Author's reply:

Many thanks for your comment. The ambiguity has been clarified in the revised manuscript. Please see the revised manuscript.

- ✓ R is the radius of the wheel and F_{ti} is the traction force of each wheel

Comment 2: Revise incorrect expressions in the text, such as "(31)" in "By substituting equation (14) into equation (31) …", "(3.4)" in "The calculation of the external torque in the first layer of the proposed control system, along with the corresponding longitudinal force distribution and extraction of the desired slips for each wheel, will be presented in Section (3.4).".

Author's reply:

Many thanks for your comment. Revision is applied in the revised manuscript based on your comment. Please see the revised manuscript.

- ✓ By substituting equation (14) into equation (13)
- ✓ The calculation of the external torque in the first layer of the proposed control system, along with the corresponding longitudinal force distribution and extraction of the desired slips for each wheel, will be presented in Section (4.3).

Comment 3: Correct the format of w1 and w2 in the sentence "In the above equation, w1>0 and w2>=0 are the weighting coefficients for the rotational velocity variable and the control input";

Author's reply:

Many thanks for your comment. The format is corrected on your comment in the revised manuscript. Please see the revised manuscript.

- ✓ $J_2(M_z) = \frac{1}{2}w_1[r(t+h_2) - r_d(t+h_2)]^2 + \frac{1}{2}w_2\omega_z^2$ (30)

  In the above equation, $w_1>0$ and $w_2>=0$ are the weighting coefficients for the rotational velocity variable and the control input, respectively.

Comment 4: The character format below Equation (34) should be consistent with the character format in Equation (34).

Author's reply:

Many thanks for your comment. Necessary changes are applied in the revised manuscript. Please see the revised manuscript.

Comment 5: Provide quantitative results for the comparison of the two methods in Figure 9;

Author's reply:

Comment 6: The references are relatively old, and it is recommended to cite the latest literatures in recent years.

Author's reply:

Many thanks for your comment. Based on your comments, the literature review is updated and several recent references are added to the revised manuscript. Please see the revised manuscript.

- ✓ Li et al. (2015) have further extended the optimal torque distribution control strategy for four-independent wheel drive electric vehicles, contributing to the refinement of this cutting-edge technology (Li et al., 2015). Their approach goes beyond conventional torque vectoring by not only improving vehicle handling but also addressing critical aspects such as minimizing tracking errors, reducing actuator control energy consumption, and optimizing tire workload distribution. This comprehensive approach underscores the importance of precise torque vectoring control in achieving superior electric vehicle performance while ensuring energy efficiency and tire longevity.

- ✓ The sliding mode control, renowned for its nonlinear and robust characteristics, has garnered significant attention among researchers due to its ability to effectively handle complex control challenges (Emam, et al., 2021; Zhang, et al., 2021). Rieveley and Minaker (2007) have made notable contributions to this field by devising a control law for computing the external torque employing the sliding mode control approach, which is recognized for its applicability in various control domains (Rieveley & Minaker, 2007). In their work, they utilized a linear two-degree-of-freedom model as a foundation to implement the sliding mode control strategy, emphasizing its versatility and adaptability in achieving control objectives in different systems and applications.
- ✓ Numerous control methods have been developed for torque distribution systems, and several of these methods are examined in the following discussion. The PID controller, a classical and widely-used control structure, stands out as one of the simplest in terms of implementation (Wang et al., 2021; Ding et al., 2021). (De Novellis et al., 2014b) conducted a study focusing on designing external torque control, where they explored two distinct approaches. These approaches include pre-feedforward PID control and robust pre-feedforward PID control, both of which were investigated in their research.
- ✓ To ensure effective torque distribution, precise control of external rotational torque is essential. Direct control of wheel rotational torque is recognized as an effective approach for achieving stability and controlling lateral dynamics (Wang et al., 2021; Lee et al., 2021). Various control methods have been investigated for regulating wheel rotational torque through torque distribution systems (Gan et al., 2013; Tavasoli & Naraghi, 2011; Braghin & Sabbioni, 2010; Rubin & Arogeti, 2015; Kasinathan et al., 2015; De Novellis et al., 2014a; Mashadi et al., 2011; Marino & Scalzi, 2008; Maroonian et al., 2013). Before delving into these methods, it is important to examine the inputs and outputs of different torque distribution strategies. While many researchers have focused on wheel rotational speed as a control variable (Gan et al., 2013; Tavasoli & Naraghi, 2011), others have explored lateral slip angle (Braghin & Sabbioni, 2010) or a combination of both variables (Rubin & Arogeti, 2015). Some researchers have designed controllers with the objective of minimizing both longitudinal and lateral wheel forces (Kasinathan et al., 2015). Furthermore, certain studies have incorporated physical limitations of the propulsion system, such as the maximum torque output, to enhance controller performance (De Novellis et al., 2014a).