

POINT BY POINT DISCUSSION ON HOW THE REVIEWERS' COMMENTS AND CONCERNS WERE ADDRESSED

REVIEWER # 3

- a. **Reviewer Comment:** “This paper tries to convert the Maxwell equation into the compressible inviscid Navier-Stokes equation when the magnetic field satisfying a generalized Beltrami condition. It is an interesting paper but I have lots of concerns/questions:
Author Answer: Thank you. This summary of my paper is indeed correct.
- b. **Reviewer Comment:** “The most concern is that the author linked the current to the vector potential, $\mathbf{J} = \rho_q \mathbf{A} / \beta_q$ from equation (5). The current describes how the media response to the electromagnetic fields and thus is an independent variable in the Maxwell equations. It shouldn't be constructed using the fields (the vector potential) without know the media property. In other words, the author considered a rather limited case satisfying $\mathbf{J} = \rho_q \mathbf{A} / \beta_q$.”
Author Answer: The medium properties are known as indicated for example in equation (1) defining β_q in terms of the mass density and charge density of the medium, also equation (4) introduces the permittivity ϵ_o , another property of the medium, and equation (5) introduces the speed of light in vacuum, another property of the medium (in this case the free space). Extending the latter to other media is standard procedure in electromagnetics, and there is no loss of generality in the choice made in equation (5) by linking the current density to the vector potential in the form $\mathbf{J} = \rho_q \mathbf{A} / \beta_q$, and eventually linking it to an electric charge velocity.

There are also many other problems:

1. **Reviewer Comment:** “The reference numbers and the citations in the manuscript don't match for Refs. 11-20, for example, Lamb's paper is numbered [11] but cited as [12] on top of page 2.”
Author Answer: The reference list was corrected to have the citations in the text linked correctly to the references. **Thank you!**
2. **Reviewer Comment:** “Above the equation 2, the electromagnetic momentum density should be $\rho_q \mathbf{A} / \beta_q$ rather than $r_q \mathbf{A} / \beta_q$ ”
Author Answer: The typo occurred when the journal's system converted the MSWord file to PDF. The MSWord file that I submitted (and my own created pdf) does show correctly the term as $\rho_q \mathbf{A} / \beta_q$.

REVIEWER # 3 (... continued)

3. **Reviewer Comment:** “The general vector potential satisfies $\mathbf{B} = \nabla \times \mathbf{A}$ rather than $\mathbf{B} = -\nabla \times \mathbf{A}$ in equation 2.”

Author Answer: The vector potential typically used in electromagnetics for the solution of Maxwell equations is indeed defined by $\mathbf{B} = \nabla \times \mathbf{A}$. However, this choice is not unique. Defining it as $\mathbf{B} = -\nabla \times \mathbf{A}$ produces identical results for $\mathbf{B}$ and $\mathbf{E}$, and the negative sign is needed in order to convert the Maxwell equations into the inviscid compressible Navier-Stokes equations consistently, subject to the generalized Beltrami condition.

4. **Reviewer Comment:** “Above equation 3, “equation (1) produces the Gauss law ...”. It should be equation (2). The same for (1) above equation (20).”

Author Answer: The text was corrected to address the reviewer concern. It now shows correctly as equation (2) and not (1) in both places. **Thank you!**

5. **Reviewer Comment:** “Below equation 3, “because the curl of the divergence of any vector is always zero”. It should be “the divergence of the curl of any vector”.

Author Answer: The text was corrected to address the reviewer concern. It now shows as: “because the **divergence of the curl of any vector is always zero.**” **THANK YOU!**

6. **Reviewer Comment:** “All the dot product sign after equation 11 shows as square rather than dot.”

Author Answer: The typo occurred when the journal’s system converted the MSWord file to PDF. The MSWord file that I submitted (and my own created pdf) does show correctly the dot product.

REVIEWER # 3 (... continued)

7. **Reviewer Comment:** “Below equation (20), the author claims that “in all these cases term $\mathbf{A} \cdot \mathbf{A}$ in equation (9) and (10) vanishes.” This is true for the generalized Beltrami condition (c) but I don’t see how it is the case for condition (a) and (b). “

Author Answer: The claim stated in the paper refers to the term $(\mathbf{A} \cdot \nabla) \mathbf{A}$ and not $\mathbf{A} \cdot \mathbf{A}$. The text was corrected to indicate that: “In all these cases the term $(\mathbf{A} \cdot \nabla) \mathbf{A}$ in equations (9) and (10) vanishes, except for its effect on the reduced pressure term in equation (12).” **THANK YOU!**

Since based on equation (11) $(\mathbf{A} \cdot \nabla) \mathbf{A} = \frac{1}{2} \nabla(\mathbf{A} \cdot \mathbf{A}) - \mathbf{A} \times (\nabla \times \mathbf{A})$ and since based on equation

(2) $\mathbf{B} = -\nabla \times \mathbf{A}$ the former equation becomes: $(\mathbf{A} \cdot \nabla) \mathbf{A} = \frac{1}{2} \nabla(\mathbf{A} \cdot \mathbf{A}) + \mathbf{A} \times \mathbf{B}$ then

(a) if $\mathbf{B} = -\nabla \times \mathbf{A} = 0$ then $(\mathbf{A} \cdot \nabla) \mathbf{A} = \frac{1}{2} \nabla(\mathbf{A} \cdot \mathbf{A})$ and the gradient term is absorbed in the reduced pressure terms making $(\mathbf{A} \cdot \nabla) \mathbf{A}$ in equations (9) and (10) to vanish, except for its effect on the reduced pressure term in equation (12).

(b) if $\mathbf{A} \times \mathbf{B} = 0$ then $(\mathbf{A} \cdot \nabla) \mathbf{A} = \frac{1}{2} \nabla(\mathbf{A} \cdot \mathbf{A})$ and the gradient term is absorbed in the reduced pressure terms making $(\mathbf{A} \cdot \nabla) \mathbf{A}$ in equations (9) and (10) to vanish, except for its effect on the reduced pressure term in equation (12).

This proof and correction addresses the reviewer concern.

8. **Reviewer Comment:** “When derive equation (25) from equation (9), the β_q is not compatible for $\mathbf{v} = \mathbf{A}/\beta_q$ due to the different powers of $\mathbf{A}$ in equation (9) where

$$\frac{\partial \mathbf{A}}{\partial t} + (\mathbf{A} \cdot \nabla) \mathbf{A}$$

This means that the either the space or the time must absorb one β_q . Then the effective gravity should be modified to take into account β_q .

Author Answer: The text was corrected following equation (22) to reflect correctly the definition of the electromagnetic momentum as $\rho_q \mathbf{A}/\beta_q$, as follows: “... identifying the electromagnetic momentum $\rho_q \mathbf{A}/\beta_q$...”

The reviewer is correct for the general Navier-Stokes equations, however equation (25) and $\mathbf{v} = \mathbf{A}/\beta_q$ are correct as long as the generalized Beltrami condition is satisfied, i.e. the impact of the term $(\mathbf{v} \cdot \nabla) \mathbf{v}$ is only on the reduced pressure term where indeed the term involving $\nabla(\mathbf{v} \cdot \mathbf{v}/2)$ should have a factor of β_q . However, this does not make equation (25) and the definition of $\mathbf{v} = \mathbf{A}/\beta_q$ incorrect. Everything is consistent as long as the generalized Beltrami condition is satisfied.

REVIEWER # 3 (... continued)

9. **Reviewer Comment:** “For the condition (c), $(\mathbf{A} \cdot \nabla) \mathbf{A}$ is zero. The equation (25) is compressible inviscid Navier- Stokes equation with additional limit.”
Author Answer: Noted! Thank you!