Reply Letter Referee 1

Dear Editor,
thanks for the work, and many thanks to the Referees for the seriousness with which they have commented. Here is my reply to Referee 1:

1. The reference list appears to be incomplete. [...] — all references added and put into context (in the introduction).

2. Why the complex-valuedness of the Dirac matrices is considered as a "problem"? On the contrary, the use of the complex analysis represents an advanced and convenient tool which helps to solve the physical problems in a very efficient way. — even more than that, complex variables are not only useful but essential in defining spinors; what I mean was only that observables must be real. I completely re-wrote the beginning of the Introduction with this key point in mind.

3. The author uses nonstandard notation which is rather confusing. I would suggest to add a remark explaining that the matrix $\pi = \gamma_5$ *. Actually it is incomprehensible to me why one needs a new notation* π *instead of the common* γ_5*. Convincing arguments are needed.* — I have explained that the index 5 initially added to work in Kaluza-Klein models is no longer a necessity; in fact, it could be risky in all those conventions in which the signature implies that lowering the index five would also entail a change of sign. The idea is of course not that of confusing, quite the opposite. As the Referee may see in my previous papers, this point is commented several times. However, in the past other Referees have commented that I should *not* add such a statement, since one uses the notation he prefers. For example, in the above-mentioned book that the Referee 1 has suggested for me in point 1., the author, Zhelnorovich, rarely uses a notation that is conventional. After all, there are so many concepts and so few letters that non-standard notations become common. In the idea of making happy those referees who asked me not to comment and those who asked me to comment, I commented, but only in a footnote.

4. The author never explains what the matrix "L" is in (12)? My guess is that this is a Lorenz spinor transformation, which therefore is determined by 6 independent real parameters. However, this obviously contradicts eq. (23), since the right-hand side contains $7 = 1 + 6$ *independent components of the scalar* ζ *and the skew-symmetric tensor* $\zeta_{ij} = -\zeta_{ji}$*. Moreover, it is unclear how the right-hand side of (23) can contain an electric charge q. Does it mean that the matrix "L" is a function of q?* — this is a very good point, it was unwise of me not to clarify it before. I have explained it now between lines 153 and 189; it is indeed a spinor transformation (also explained at line 106): the difference between L and S is also explained at page 5. I discuss, in the same page 5, why, in being the product of a Lorentz times a gauge transformation, it has 6+1=7 parameters, although one is redundant. This discussion about redundancy and the reduction from 7 to 6 parameters is also made around the old equation (23), now equation (21), in lines 205-207. More mathematical details about this redundancy have been given through equations (27) and in lines 221-223 and 233-238. The right side of old (23), now (21), contains an electric charge q because of the convention we choose to simplify the definition (22). This is also now explained at lines 208 to 210.

5. In sec. 3, eqs. (29)-(51) contain an object W which is described as an axial vector of torsion. Does it mean that the connection in eqs. (21) and (22) also contains the non-Riemannian torsion piece? — explanation given now in the first paragraph of section 2.2.

6. Eqs. (41)-(77) contain an object "P" which is described as the 4-momentum. How this object "P" is related to the spinor wave function? — P is related to the wave function in terms of what is now equation (24). The fact that this is the momentum is easy to see when neglecting beta and the module and also R: multiplying what remains by i one gets $i\nabla_i\psi = P_i\psi$ showing that P is indeed the (dynamical) momentum. A more general way to prove this is possible and it has been done in reference [26], now cited for this purpose. Another clarification is in lines 280-285.

7. Eq. (66) is the same as (54), whereas eq. (67) coincides with (57). Why such a duplication is needed? — instead of writing new equations and citing old ones, I have preferred to write them all grouped together for clarity and completeness.

8. On page 7, the author mentions the "unitary gauge" (in particular, on line 186, we read that "we may go in the unitary gauge"). What is it? How one can "go into a unitary gauge"? Recalling that physics is an experimental science, the author should be able to explain to experimenters how to "go into a unitary gauge". Which experimental apparatus helps to do this? — the unitary gauge is the one where the spinor is at rest and with spin aligned along the third axis (this is now explained, and repeated several times along the text). We have also specified that we have called it like that because of a mathematical analogy with the Standard Model (in lines 240-242). From this one may deduce that we go in the unitary gauge when the spinor is kept still and in a magnetic field aligning spin in a given direction, although I am not an experimentalist and would not be able to tell how this can be done in practice. We would like however to specify that the expression "going into unitary gauge" should not be read as the material moving into such a gauge, but as the choice of gauge needed for some mathematical simplifications, in exactly the same way in which we "go in the unitary gauge" in the Standard Model, when we want the Goldstone bosons to be "eaten" by the gauge fields, without actually meaning that we are materially moving in that gauge, or even less that with this choice the gauge fields are really eating the Goldstone fields. It was my understanding that in the Standard Model

these were metaphorical expressions used to convey ideas in a quick manner, and I was simply borrowing them.

9. Readers may wonder why (30)-(51) is a "hydrodynamic representation"? — this is now explained (in the second paragraph of Intro, in the first of sec. 2.1.1., between lines 286-296 of page 8).

10. More explanations are needed in Sec. 4 concerning 19 formulations. I wonder, in particular, are these formulations equivalent among themselves or not? If they are, then the Abstract on page 1 is totally misleading, because on line 2 and 4, the author talks about "19 different sets" and "19 different ways". So, are they different or equivalent? — yes, they are all equivalent, as is proven in section 3 now. I also have changed abstract and Intro accordingly trying to specify better that by "19 sets" and "19 ways" I mean the 19 systems of field equations that are obtained in terms of 19 ways to re-arrange them covariantly. It should be clearer now.

11. In order to make the paper self-contained, the author should explain what are "Fierz identities" and "Gordon decomposition"? Please explain: Identities between which objects? Also: Decomposition of which objects are meant? Perhaps, additional references to the corresponding literature sources would be helpful. — I have taken away the reference to Fierz but still explained what are those identities (line 125). I have explained what are the Gordon identities (lines 266-269, lines 273-274, lines 280-285). A reference of QFT was added for this purpose.

12. What is Σ^{ij} in eq. (8)? Please define this object. — Object defined in equation (2).