

Response to Reviewer 1

June 3, 2023

The authors express their thanks for the work of the Reviewer. Our responses to his/her suggestions/questions are below:

1. In this paper, the authors address an interesting problem in statistics about the existence and uniqueness of MLEs in some certain circumstances. I find this work interesting and enjoy reading it. Moreover, the results that the authors obtained are believable. So, it is definitely worth publishing. However, a few aspects of the paper can be improved via a revision. See my comments below for specific suggestions.
[Thank you very much for your opinion and suggestions.](#)
2. Add at least two Mathematics Subject Classifications 2020 to the paper.
[Three "Mathematics Subject Classifications 2020" have been added.](#)
3. Some additional reference to increase paper's readability. Note that adding the following references are optional and my final decision is independent from adding these references.
 - 1- Graßhoff U, Schwabe R. Optimal design for the Bradley–Terry paired comparison model. *Statistical Methods and Applications*. 2008 Jul;17(3):275-89.
 - 2- Courcoux P, Semenou M. Preference data analysis using a paired comparison model. *Food quality and preference*. 1997 Sep 1;8(5-6):353-8.
 - 3- Cattelan M. Models for paired comparison data: A review with emphasis on dependent data.
 - 4- Wickelmaier F, Schmid C. A Matlab function to estimate choice model parameters from paired-comparison data. *Behavior research methods, instruments, and computers*. 2004 Feb;36:29-40.
 - 5-Mahdizadeh M, Zamanzade E. Smooth estimation of the area under the ROC curve in multistage ranked set sampling. *Statistical Papers*. 2021 Aug;62(4):1753-76.
 - 6-Zamanzade E, Mahdizadeh M, Samawi HM. Efficient estimation of cumulative distribution function using moving extreme ranked set sampling with application to reliability. *ASTA Advances in Statistical Analysis*. 2020 Sep;104(3):485-502.

Thank you. The suggested papers have been inserted in the current paper.

4. Not clear!! You mean a simulation with large number of repetitions or a simulation with variety choices of model parameters.

We mean: large number of repetitions of large variety of parameters (m , d , number of comparisons) for different number of objects. Only a small part of the results are included in the paper. The sentence has been corrected.

5. To save the journal space, you can move the proof to the online supplementary material if it is applicable.

As Axioms is an electronic journal and this paper is not too long, we would like that you and the Editor allow the proof in Appendix.

6. The figure sizes 4-5 are not compatible with Figures 2-3.

The figures have been changed into others produced by GraphViz software, and the sizes are already equal.

7. It would be useful if you provide the necessary codes for reproducing the results publicly available.

The program is long and it has not been commented yet. We have rather detailed the simulation procedure.

8. It would be extremely useful if you add some practical applications for your proposed procedures.

Two sports examples have been inserted into a new section.

The authors thank again the useful suggestions of the reviewer. We hope that the paper has been improved appropriately by the modifications.