Response to Reviewer 4 Comments

Dear Reviewer 4,

Your suggestions and comments are very valuable for us to improve the manuscript. Our responses to your comments are listed one-by-one in the following:

Point 1: Page 3: For (2), (3), etc., perhaps the authors should state that the summation convention is being used, i.e., a sum is performed over terms with a doubly repeated subscript (and also point out where a doubly repeated subscript does not indicate a sum), for clarity.

Response 1: These are very good concerns. In geophysics community, these expressions are actually common for elastic wave physics. Most of geophysicists should understand these equations. In the revised manuscript for equations A4 and A5, we have added the statement of "and summation convention is being used for the subscripts of these displacement wavefields.", which should be enough for clarity.

Point 2: Page 3: In (7) perhaps, for clarity, use brackets on the right side to indicate if this is the derivative of u_iu_i or just the derivative of u_i multiplied by u_i. Perhaps the other equations in the paper should also be looked at for this, with clarity in mind.

Response 2: These are very good suggestions. In revised manuscript, we have added bracket in equation (A6) and other questions for clarity in the revised manuscript.

Point 3: Page 5: Is the sentence immediately above Figure 1 a general statement, i.e., is it true for all cases, or does it apply only in this specific case?

Response 3: This is also a very good concern. We designed this objective function for reducing the influences of wrong source wavelet with arbitrary errors. The wrong source generated by applying phase rotation to the correct wavelet is the optimal case. As suggested by other reviewers, in the revised manuscript, we generate a new wrong source wavelet with phase and amplitude changes (not just phase rotation). The method still works effectively.

However, in some cases, this method does not work well. We have added some discussions about this issue in the discussion section.

Point 4: Page 6, eq. (14)-(25): The authors introduce a new envelope traveltime objective function and a new adjoint source in the theory section and use them to demonstrate the improvement. But how new are these results? What is different about their new envelope function when compared to those used in previous work? The differences between the envelopes of observed and synthetic traces have been used by others in previous work, according to the authors. In fact, the authors

point this out and cite several references. Or has the envelope method never been used before to reduce waveform distortions? Is that what is new here? Some further clarification on how different their work is from previous work might be useful.

Response 4: These are very good concerns. As we described in the manuscript, the envelope difference objective function has been widely studied and applied for reducing the cycle-skipping problem in full-waveform inversion. However, as far as we know, previous researchers never investigate the capability of envelope for reducing phase and amplitude distortions caused by inaccurate wavelet. **Using the cross-correlation based envelope traveltime objective function to reduce the influences of source-velocity trade-offs is innovative contribution of this study**. We never present related researches in previous publications.

For clarification these innovations, in the revised manuscript, we have added the following statements in the introduction section:

"Using cross-correlation based ET objective function for reducing the source-velocity trade-offs has not been investigated by other researchers and represents innovative contribution of this study."

Point 5: Page 9, Figures 4-8: If I understand correctly, it seems to me that it is predictable that correlating the envelopes of the observed and synthetic traces gives better results than correlating the observed and synthetic traces themselves, because the envelopes of the traces resemble each other more than the traces themselves resemble each other. Is this correct?

Response 5: This is very good question. Yes. You understand these results correctly. In conventional cross-correlation based traveltime objective function, when a wrong source wavelet is used in forward modeling, the calculated traveltime shifts are inaccurate leading to inaccurate velocity inversion results. However, when using envelope cross-correlation objective function, envelope reduces the waveform distortions caused by inaccurate source wavelet. The calculated traveltime shifts and velocity inversion results are still accurate.

Point 6: Pages 12-14: If I understand correctly, in the Vibroseis field data, it is possible to directly calculate the correct (Klauder) wavelet. Therefore, the ET approach is not needed. Figures 12a and 13a give the best possible results. Is this correct? Are the authors then suggesting that the ET approach be used only for other non-Vibroseis data? Would the ET method work just as well for other data?

Response 6: These are very important concerns for this study. Yes. You understand this field data application example correctly.**The objective of this example is to verify the effectiveness of our method for practical noisy data set.** As we mentioned in the manuscript, in this practical application example, the Klauder wavelet is actually known. Figures 12a and 13a give the best results using the correct wavelet. We assume that if a wrong source wavelet is used, traditional method can not obtain reliable inversion results. However, if the new envelope traveltime objective

function is used, reliable inversion results can still be obtained. The results are close to that obtained using traditional traveltime objective function with correct wavelet.

From these results, we suggest that our method can also be used in other non-Vibroseis data with unknown source wavelet. In the revised manuscript, we have added the following statements for clarification:

"This field data application example also suggests that our new method can work effectively for other non-Vibroseis data set with unknown source wavelet."

Point 7: Pages 18-19, Discussion and Conclusions: As the envelopes of the wavelets can be thought of as "smoothed" or "averaged" versions of the wavelets, it seems almost counter-intuitive that the ET method should give better results by smoothing out waveform distortions (because of the loss of information produced by smoothing). Are there cases where the ET method would not work as well? Do the authors see any disadvantage to the ET method because of this "smoothing"?

Response 7: These are very good questions. It is true that smoothing a signal can reduce its waveform variations to some extent. However, we think that envelope of a signal is not equivalent to a "smoothed" or "averaged" version of the signal. We think they are different. Envelope measures the instantaneous amplitude of a signal meaning that its values are all positive. However, smoothing a signal can not make the signal's polarity reversed.

There are some cases where ET method doesn't work very well. Other reviewers also mention this problem. We design this new algorithm and hope that it works for most cases. We have tested different cases about this issue. We found that when the shapes of the two wavelets are quite different, this method doesn't work well. For examples, if the source wavelets contains many irregular side lobes or the correct and wrong source wavelets have different frequency bands. In the discussion section, we have added the some statements about this issue

One disadvantage of ET method is that compared to waveform-difference objective function, detailed velocity structures can not be resolved.