

Temporal-spatial Mechanisms of Soil Subjected to Seepage-induced Erosion

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Abstract: Change the hydraulic condition can reduce the effective stress of the soil and result in the migration of particles with a threat to the stability of infrastructures, which would be local affected. We employed the coupled computational fluid dynamics (CFD)-discrete element method (DEM) to study the impact of seepage duration. Increasing the fine contents would reduce the particle velocity and drag force for enhancing the erosion resistance. The representative volume elements (RVE) were selected to observe the local geometry and material behaviour as the erosion progressed. Seepage erosion would reduce the stability of the structure at different positions of the sample with various fine contents. Severe seepage flow reduced the peak shear strength significantly of the eroded soils for the interactive-underfilled soil, in particular for the downstream side. However, the interactive-overfilled specimen reduced the peak shear strength on the upstream side. Results indicated that the proposed weighted clustering coefficient is a better index to capture the shear strength of the eroded soil matrix, with a high magnitude corresponding to a high peak shear strength. The different local material behaviour may lead to differential settlements and associated catastrophic consequences subjected to a long-term seepage, which should draw special attention.

Keywords: seepage erosion; fine content, local geometry, weighted clustering coefficient, mechanical behaviour, particle-fluid interaction force, CFD-DEM

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1. Introduction

In nature, the hydraulic condition may vary slowly or rapidly as rainfall or human activities. The change in the hydraulic gradient can trigger the detachment of particles inside porous materials, which refers to the internal erosion phenomenon. Sterpi [1] stated that the effect of gradual erosion and transport of fine particles due to a severe seepage flow could not be ignored when working in an urban area, which is one of the most common reasons for failure in dams, roads, and railways. The water-retaining structures such as dikes and dam embankments are typically constructed on complex foundations, which should draw attention to the internal erosion.

Foster et al. [2] summarized the incidents of dam failure taken from ICOLD and found that 46% occurred due to piping through the embankment. Terzaghi [3] first performed experiments on heave-type piping failure around a sheet pile wall and proposed a safety factor formula against the hydraulic heave phenomenon. The heave phenomenon is a special case of the seepage phenomenon, which occurs when the hydraulic gradient is high enough to vanish the effective mean stress. Generally, it is accepted that the onset of the seepage phenomenon is highly dependent on the ratio of finer fraction and the particle size distribution [4-7]. Bendahmane et al. [8] found the erosion rate is higher for a sample with lower content of kaolin and proposed that a high content of clay contributed to reducing the flow density and increasing the internal erosion resistance. Similar results were observed by Sato and Kuwano [9] and Xie et al. [10]. The clay fractions preferentially migrate resulting in a great amount of finer fractions of the eroded soil.

It is noticed that the detachment of soil particles affects both the micro and macro-structural behaviour of soil [11]. Numerous works have been reported on the mechanical behaviour of eroded specimens under different stress paths with the development of the experimental apparatus [7; 12-14]. Wang et al. [15] employed a developed stress-controlled apparatus with a cut-off wall to observe the internal erosion and found the higher deviatoric stress would produce a dilatant behaviour for loosening the soil sample with the phenomenon that occurred at a lower hydraulic gradient. Xiao and Shwiyhat [16] performed the undrained monotonic compression tests on internally eroded soils. They proposed that the suffusional soil would induce a higher peak deviatoric stress than the results of the soil without suffusion. Ke and Takahashi [12] found the loss of fines in the soil matrix resulting in a contractive volumetric strain. They proposed that the suffusion phenomenon would reduce the soil strength at the major stage of drained shearing from the monotonic compression test. Ke and Takahashi [14] found the soil-drained strength decreases after suffusion under the successive drained monotonic compression test and results in a temporary drop in stiffness at the initial stage of shearing.

In recent decades, numerical simulation is one of the commonly used approaches to understanding the mechanisms of internal erosion. Cundall and Strack [17] first developed the discrete element method (DEM) to model granular materials' behaviour with the geometrical shape. Adopting DEM, Scholtes et al. [18] proposed an approach for progressive removing the finest particles from a soil matrix while the sample was maintained under constant stresses. The concept could be used to simulate the process of internal erosion and investigate the impact on the soil properties at the microscopic scale [19,20]. Hicher [21] found the eroded soil masses could develop instability at a shear stress level lower than the critical state failure line. Hama et al. [22] studied the instability of granular materials after suffusion using the three-dimensional Particle Flow Code (PFC3D) software. They found that erosion alters the material behaviour from dilatancy to contractility. Chen et al. [23] observed similar results and proposed that the soil matrix would produce a lower peak and critical friction angle with the loss of fine particles.

The reported works mainly examined the soil strength by removing particles. However, Aboul Hosn et al. [24] found that the soil matrix would maintain the same probability density of contact forces and mechanical coordination number as non-eroded soil if the inactive particles are removed. They proposed that post-erosion mechanical behaviour cannot be described only by an increase in porosity. Moreover, most studies are devoted to the mechanical behaviour of post-eroded samples subjected to a downward or upward seepage flow. However, the erosion path would be complex concerning the hydraulic structures like earth dams and embankments, which should be position-dependent and evolve with time. The reported works on the local affected zone in the hydraulic structure are still limited, which is the main cause of differential settlements and associated catastrophic consequences [25]. This study is conducted to fill this gap by investigating the seepage impact on the internal heterogeneity of granular systems and examining the mechanical behaviour of eroded soil matrix at different locations of embankments to give some insights into the design of practical infrastructures. The paper is structured as, Section 2 elaborates on the methodology of the numerical software. The temporal-spatial observations of internal erosion on the grain transport and the geometric conditions are given in Section 3. Section 4 presents a discussion of the mechanical behaviour of the eroded soil. Finally, we conclude and draw perspectives of this work.

2. Methodology

We adopt the open-sourced CFD-DEM software to perform simulations with multi-phase media. The discrete particles were mimicked by the LAMMPS improved for general granular and granular heat transfer simulations (LIGGGHTS) software [26], and

the fluid flow was simulated by the Open Field Operations and Manipulation software (OpenFOAM) software [27].

OpenFOAM adopts the finite volume (FV) method to discretize the complex geometries into cells control volumes. The following continuity equation is assumed to hold for satisfying the conservation of mass on each control volume:

$$\frac{\partial(\alpha_f)}{\partial t} + \nabla \cdot (\alpha_f \mathbf{u}_f) = 0 \quad (1)$$

where $\mathbf{u}_f$ and α_f denote the velocity and porosity of fluid flow respectively. ∇ is the gradient operator.

The motion of fluid flow is governed by the volume-averaged Navier-Stokes equation and solved in conjunction with equation (1),

$$\frac{\partial(\alpha_f \rho_f \mathbf{u}_f)}{\partial t} + \nabla \cdot (\alpha_f \rho_f \mathbf{u}_f \mathbf{u}_f) = -\alpha_f \nabla p - \mathbf{K}_{sl} (\mathbf{u}_f - \mathbf{u}_p) + \nabla \cdot \boldsymbol{\tau} \quad (2)$$

where $\mathbf{u}_p$ refers to the averaged particle velocity. The stress tensor for the fluid phase is calculated as $\boldsymbol{\tau} = \nu_f \nabla \mathbf{u}_f$, in which ν_f is the fluid viscosity. $\mathbf{K}_{sl}$ denotes the momentum exchange with the particulate phase, which is calculated based on the particle-fluid interaction force for each cell.

For the particulate phase based on Lagrangian modelling, the linear and angular velocities for each particle are computed on the basis of Newton's second law of motion. The governing equations of translational and rotational motion of a particle with mass and momentum are as follows [26]:

$$m_i \frac{d\mathbf{v}_i}{dt} = \sum_j \mathbf{F}_{ij}^c + \mathbf{F}_i^f + \mathbf{F}_i^g \quad (3)$$

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum_j \mathbf{M}_{ij} \quad (4)$$

where $\mathbf{F}_{ij}^c$ and $\mathbf{M}_{ij}$ are the contact force and torque for the particle i with the particle j or the boundary. $\mathbf{F}_i^f$ and $\mathbf{F}_i^g$ refer to the particle-fluid interaction force and the gravitational force acting on particle i respectively. Details of the definitions of two-fluid phases, solvers for the simulations and the coupling procedures can be found in Xiao and Wang [28] and Xiao [29].

3. Observations of seepage erosion

Kenney and Lau [5] proposed that there are three necessary conditions for the occurrence of the seepage phenomenon as, (i) a primary fabric of particles to support the imposed stress; (ii) the system containing loose particles that can be moved by forces such as water flow; (iii) the sizes of constrictions in the pore network must be larger than some of the loose particles for affording the possibility of being moved. Xiao and Wang [28] have studied the hydraulic condition on the onset and development of the seepage phenomenon. This paper mainly concerns the fine contents on the influence of primary fabric and erosion resistance.

3.1. Material properties

The materials are simulated as a mixture of fine and coarse fractions to simplify the phenomenon, with radii of 0.25 and 1 mm. Three samples are prepared by the fine contents (FC) of 30%, 40% and 60%, with the information of particles presented in Table 1, which are denoted as FC30, FC40 and FC60, respectively. According to Frias Lopez et al. [30], the specimen of FC30, FC40 and FC60 could be defined as the interactive-underfilled, interactive-overfilled and overfilled soil skeleton, respectively. Simula-



tions are conducted by the Hertz contact model, with the parameters for the simulations given in Table 2.

Table 1. The information of specimens.

	FC30	FC40	FC60
Mass ratio (fine/coarse)	3:7	4:6	6:4
Number of particles	446,995	560,175	817,283

Table 2. Properties of particles.

Young's modulus	Radii of particles	Friction coefficient	Poisson's ratio	Restitution coefficient
600 MPa	0.25 mm 1 mm	0.5	0.45	0.1

3.2 Numerical model set-up

This study prepares samples with particles deposited into the container under the self-gravity condition at an acceleration of $9.81m/s^2$. After the system achieves the quasi-static condition, those particles located in the region of the retaining wall are deleted to import the boundary. The wall, with a dimension of 0.01 m in the x-direction, is set in the centre of the system at a height above 0.03 m. The heights of deposited layers are 0.035 and 0.05 m for the left and right sides of the boundary, respectively. Then, the sample is submerged into the fluid flow under a saturated condition to conduct the erosion test, which is performed by defining different hydraulic heads on the two sides of the boundary. Based on the work of Xiao and Wang [28], this study examines the erosion process at a high hydraulic gradient. The left and right sides of the wall are determined as the downstream and upstream sides, with the water table set at 0.03 and 0.09 m, respectively. The steps of the coupling interval are set at 100 to reduce the computational effort. Figure 1 displays the initial configuration of the specimen.

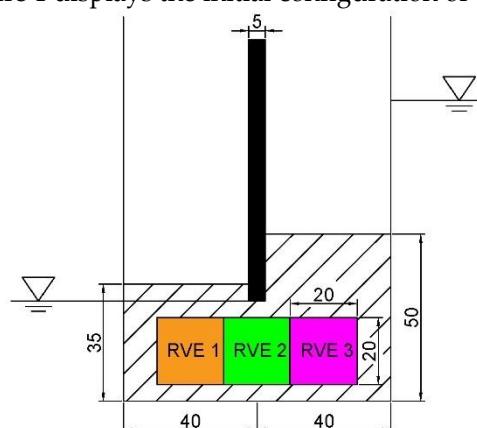


Figure 1. Initial configuration of the specimen.

3.3 Particle velocity

The seepage phenomenon occurs as the water flows through the voids inside the soil skeleton from a high hydraulic condition to a low hydraulic condition, which results in the agitation of particles. Figure 2 depicts the behaviour of particle velocity at $t=0.01s$ to observe the fine contents on the influence of the onset of seepage erosion. Our results agree with the previous reports that increasing the fine contents would enhance the critical hydraulic gradient of the system [31–32]. Hence, the specimen FC60 reduced the de-

tachment of particles with the higher particle velocity only appearing around the retaining wall.

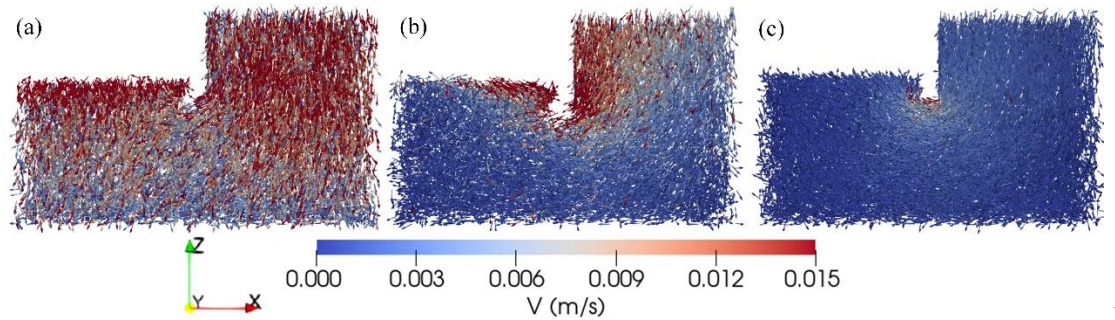


Figure 2. Spatial distribution of particle velocity for specimen (a) FC30; (b) FC40 and (c) FC60.

Taking the specimen FC40 as an example, Figure 3 and Figure 4 portray the temporal-spatial distribution of water flow and particle velocity during seepage, respectively. In Figure 3, the blue colour refers to the air phase and the red colour denotes the water phase. It is noticed that the water flows would reduce the hydraulic gradient during the seepage process. In such a case, the peak velocity was observed at the initial stage in Figure 4 and decreased during seepage.

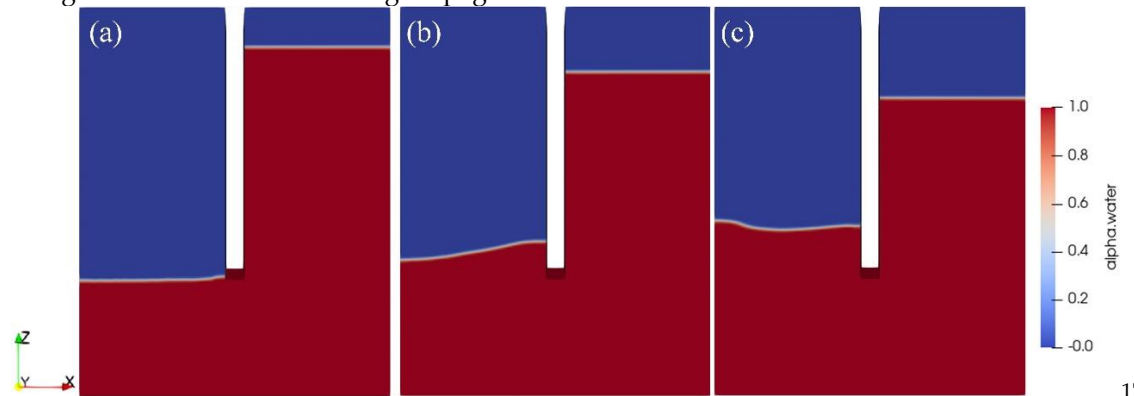


Figure 3. Evolution of water flow at $t=(a)$ 0.01 s; (b) 0.1 s and (c) 0.2 s.

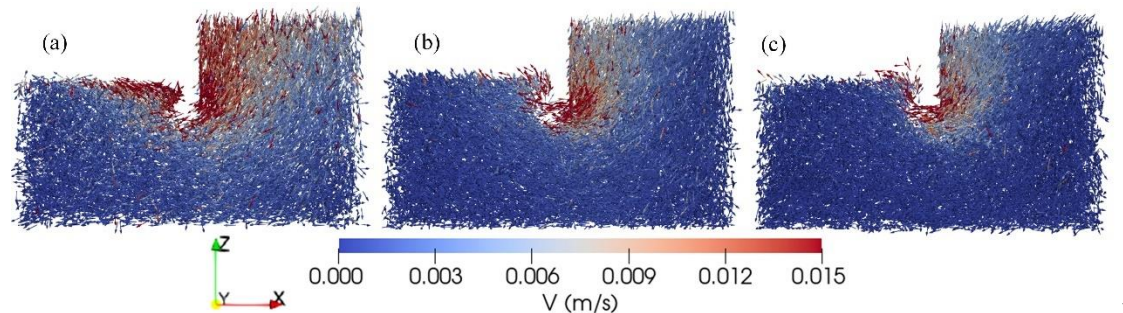


Figure 4. Evolution of particle velocity of specimen FC40 at $t=(a)$ 0.01 s; (b) 0.1 s and (c) 0.2 s.

3.4 Drag force

Concerning the seepage process, the water flows would exert a drag on the soil properties in the direction of its motion. This study concerns the Di Felice drag force (F_{dp}) as the particle-fluid interaction force, with the expressions as [33]:

$$F_{dp} = K_{sl} V_p \cdot |\mathbf{u}_f - \mathbf{u}_p| \quad (5)$$

$$\text{in which } K_{sl} = \frac{3}{4} C_d \frac{|\mathbf{u}_f - \mathbf{u}_p|}{d_p} \varepsilon^{1-\chi}, \quad \chi = 3.7 - 0.65 \exp \left[-\frac{(1.5 - \lg \text{Re}_p)^2}{2} \right] \cdot V_p \quad 185$$

is the volume of a particle, C_d denotes the drag coefficient, estimated as 186

$$C_d = \left(0.63 + \frac{4.8}{\sqrt{\text{Re}_p}} \right)^2. \quad \text{Re}_p \text{ refers to the Reynolds number with the expression as } 187$$

$$\text{Re}_p = \frac{|\mathbf{u}_f - \mathbf{u}_p| d_p}{\nu_f}. \quad 188$$

Taking $t=0.01$ s as an example, Figure 5 displays the spatial distribution of drag force. It can be seen that the higher drag force is located adjacent to the boundary for each specimen. The yellow line in the graph represents the flow path inside the soil matrix. In analogy to the behaviour of particle velocity, the results indicated that increasing the fine contents would reduce the magnitude of drag force and narrow the flow path. The specimen FC30 produces a wide flow path to the bottom of the system. However, the specimen FC60 only forms the flow path around the retaining wall. 189
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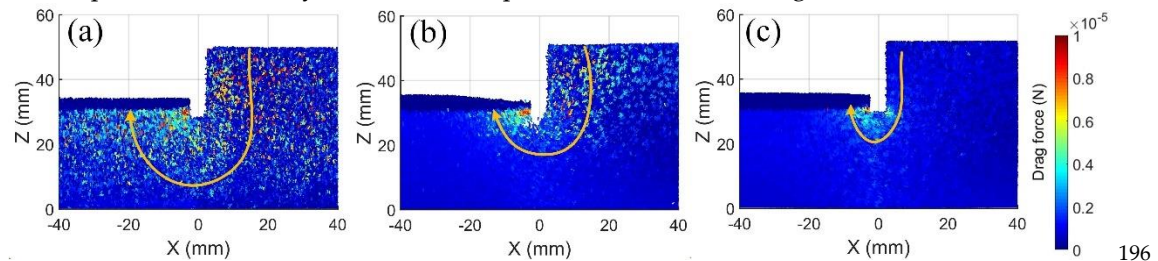


Figure 5. Distribution of drag force at $t=0.01$ s for specimen (a) FC30; (b) FC40 and (c) FC60. 196
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3.5 Local observations of seepage erosion 198

The spatial distribution of particle velocity and drag force implies that the seepage flow would induce different flow paths and different heterogeneous conditions at different positions, which may influence the soil behaviour during seepage. Hence, we selected three representative elementary boundaries (RVE) with the size of 0.02 m to observe the characteristics of the local geometry, as shown in Figure 1. The position of elementary boundaries is located in the intermediate foundations to reduce the boundary effect, with the bottom at 0.005 m. 199
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3.5.1. Rattler contents 206

It is believed that inactive fine particles (sitting loose in the voids with minor participation in the force chains) are the most vulnerable to suffusion [34]. Hence, rattlers in a state with zero or one contact should be easily eroded during the seepage process. Figure 6 depicts the evolution of rattler contents for the three specimens. The water flow would transfer the eroded particles out of the soil skeleton, which reduces the rattler contents during seepage for each sample. The specimen FC30 has a lower erosion resistance, which induces a significant change of rattlers and contains the highest proportion at the initial stage. However, the overfilled specimen FC60 enhances the connectivity between particles and involves the lowest number of rattlers, with a higher content appearing in the element RVE_C. The results indicated that the region adjacent to the boundary highly relates to the onset of seepage erosion. 207
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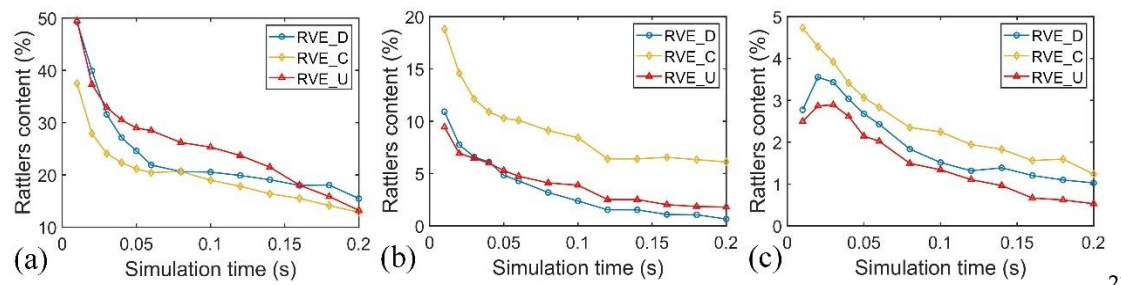


Figure 6. Evolution of rattler contents for specimen (a) FC30; (b) FC40 and (c) FC60.

3.5.2. Eroded particles

The eroded particles refer to the particles that experience a higher particle-fluid interaction force than the body force and internal-particle contact force. Hence, we estimate the content of eroded particles inside the elementary boundary to better understand the seepage impact, as shown in Figure 7. Due to the reduction of hydraulic gradient, the number of eroded particles is also reduced. As expected, the specimen FC60 shows the lowest content of eroded particles. The onset of detected particles mainly occurs around the boundary and on the downstream side, in particular for the element RVE_C according to the results of the specimen FC40 and FC60. For the specimen FC30, the high hydraulic condition would transfer particles out of the system on the downstream side. In such a case, the element RVE_D shows higher eroded contents at the initial of seepage erosion.

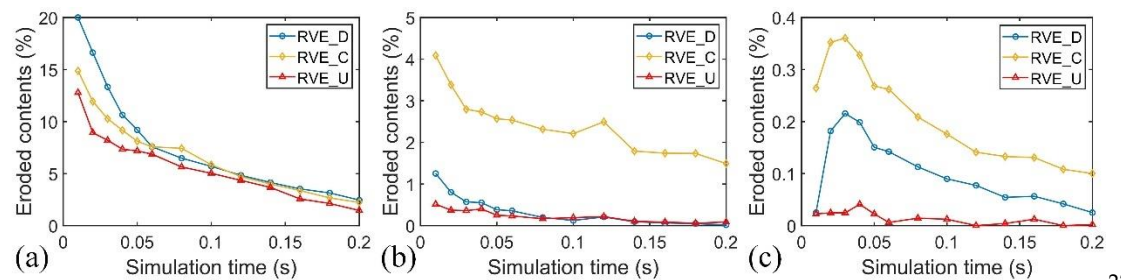


Figure 7. Evolution of eroded contents for specimen (a) FC30; (b) FC40 and (c) FC60.

3.5.3. Coordination number

Coordination number (CN) is a basic index for illustrating particle connectivity and investigating the contact network, indicating the average number of contacts a particle has. Thornton [35] defines the mechanism average coordination number without the consideration of rattlers. The expression of mechanism average coordination number (Z_m) is as,

$$Z_m = \frac{2N_c - N_p^1}{N_p - N_p^1 - N_p^0} \quad (6)$$

where N_c refers to the number of contacts, N_p is the number of particles. N_p^1 and N_p^0 denote the number of particles with one and zero contacts, respectively.

Figure 8 displays the evolution of the mechanism and average coordination number for each specimen. The results are consistent with the characteristics of rattler contents. A higher proportion of higher rattlers corresponds to a lower mechanism average coordination number. Specimen FC60 shows the highest magnitude among the samples, while the lowest magnitude occurs for specimen FC30. The results indicated that the seepage flow would enhance the particle connectivity, especially for the interactive-underfilled soil matrix.

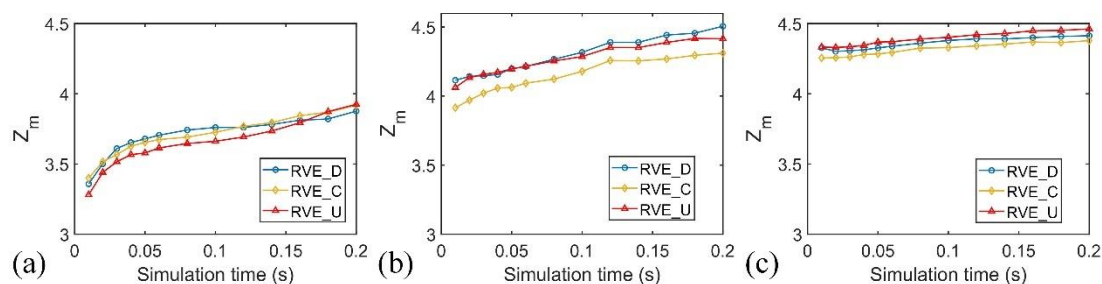


Figure 8. Evolution of Z_m for specimen (a) FC30; (b) FC40 and (c) FC60.

3.5.4. Geometry

Changes in the eroded particles and particle connectivity would lead to a different particle contact network. A detailed study of the complex networks could give a quantitative understanding of the geometry condition of the granular system. Papadopoulos et al. [36] have reviewed different methods for characterizing and quantifying the topological and geometrical organization of the granular network, such as the node degree, walks and paths. In this study, we based on the network concept of the cycle to understand the structural organization. The 3-cycles of particles arranged in the form of a triangle are the simplest motifs in complex networks, which would result in frustrating rotations and a stable condition. Taking the element RVE_C as an example, Figures 9–11 depict the distribution of the 3-cycles connections inside the soil skeleton at the initial state. It can be seen that the number of triangles increased with the increase of the initial fine contents, implying a more stable condition and a higher internal erosion resistance. The triangles are similar during seepage for specimen FC60, which are increased for specimen FC30 and FC40. Moreover, according to Figure 7, the seepage erosion induces a higher number of floaters around the retaining wall for the specimen FC40, which thus produces a sparse contact network near the top of the RVE boundary.

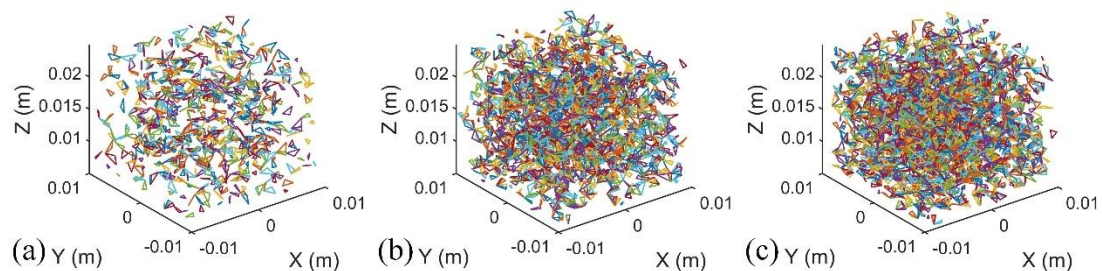


Figure 9. 3-cycles connection for the specimen FC30 at $t =$ (a) 0.01 s; (b) 0.1 s and (c) 0.2 s.

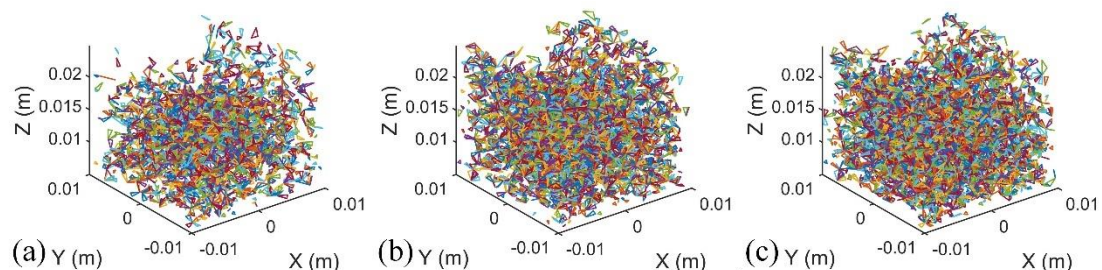


Figure 10. 3-cycles connection for the specimen FC40 at $t =$ (a) 0.01 s; (b) 0.1 s and (c) 0.2 s.

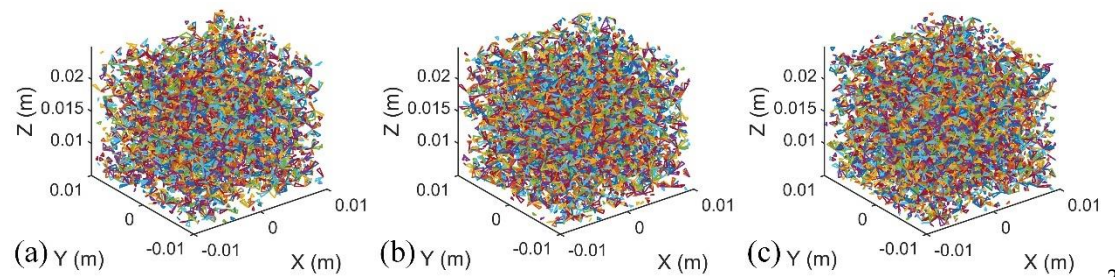


Figure 11. 3-cycles connection for the specimen FC60 at t = (a) 0.01 s; (b) 0.1 s and (c) 0.2 s.

For a given contact network, Newman [37] proposed a fundamental quantity named as clustering coefficient to measure the density of triangles. Watts and Strogatz [38] gave the definition of the clustering coefficient (C_i) at a vertex i ,

$$C_i = \frac{2t_i}{k_i(k_i - 1)} \quad (7)$$

where t_i is the number of triangles attached to the vertex and k_i is the degree of the vertex.

The connectivity between internal-particles contains the information of the contact force. Hence, the weighted clustering coefficient, obtained by the vertex strength, could provide more significant information of the complex networks. The quantity of the weighted clustering coefficient (C_i^w) at a vertex i is measured as,

$$C_i^w = \frac{1}{s_i(k_i - 1)} \sum_{j,h} \frac{(w_{ij} + w_{ih})}{2} a_{ij} a_{ih} a_{jh} \quad (8)$$

where $s_i = \sum_j a_{ij} w_{ij}$. From the vertex i to the vertex j , a_{ij} equals to 1 if the connect exists, otherwise the coefficient equals to 0. w_{ij} refers to the weight of the edge connect, which refers to the normal contact force in this study. The clustering coefficient ranges from 0 to 1. A higher quantity of C_i means a closely packed structure with more 3-cycles whereas a lower value corresponds to the system in a loose condition with less connectivity. In the entire network, the average weighted clustering coefficient (C^w) is estimated as,

$$C^w = \frac{\sum C_i^w}{N} \quad (9)$$

in which N is the number of particles.

Figure 12 portrays the evolution of C_w during seepage for different specimens. As expected, the interactive-underfilled specimen FC30 shows the lowest value of C_w among the three samples. The seepage impact transmitted the fine particles and enhanced the particle connectivity during the seepage process, which increases the clustering coefficient. It is interesting to observe that the specimen FC40 generates a higher clustering coefficient than the specimen FC60, which may depend on the work of the contact force. Desu and Annabattula [39] found that the larger particles have a higher probability of experiencing stronger contact forces than the smaller particles. Hence, though the specimen FC60 forms an intensive contact network, the high proportion of interactions of fine particles would carry a lower contact force, which reduces the weighted clustering coefficient. The graphs indicate that the element boundaries at different positions experience different heterogeneous conditions, which may impact the constitutive material behaviour.

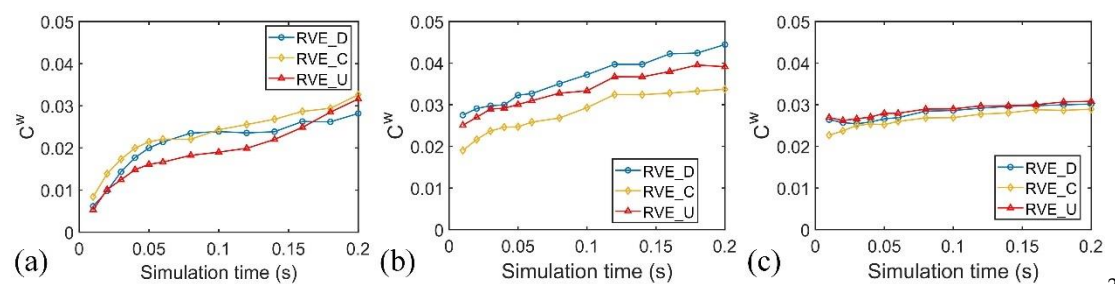


Figure 12. Evolution of C_w for the specimen (a) FC30; (b) FC40 and (c) FC60.

4. Constitutive material behaviour and discussions

The progressive observations of internal structure indicate that the seepage flow would alter the particle arrangement and the network of the system, which may impact the strength behaviour of the system. It is thus necessary to investigate how the seepage process affects the local constitutive behaviour of soils inside the specimen. In this section, we conduct the triaxial shear simulations with the three RVE boundaries for the non-eroded specimen and the temporal eroded specimen at $t=0.1$ s and $t=0.2$ s to characterise the seepage impact on the constitutive mechanical behaviour.

4.1. Numerical model set-up

Before the triaxial shear test, samples are isotropically consolidated to the confining pressure of 500 kPa. Similar to the DEM modelling of the soil matrix reported in the literature [40], the gravity of the solid particles is removed in the consolidation stage to avoid the initial heterogeneous condition. During simulation, the granular media should be under a quasi-static condition. A dimensionless parameter (I), called the inertial number, is adopted to identify the equilibrium condition in this study, with the expression as follows,

$$I = \dot{\epsilon} d \sqrt{\frac{\rho}{p'}} \quad (10)$$

where $\dot{\epsilon}$ is the strain rate, d is the mean size of grains in the assembly. ρ and p' refer to the material density and the mean effective stress, respectively. The parameter has been widely used to describe the state of granular materials in a steady flow, with the quasi-static condition identified as $0 < I < 10^{-3}$ [41]. For the 3D DEM simulation, Lopera Perez et al. [42] found the quasi-static regime of geotechnical element testing achieved with $I \leq 7.9 \times 10^{-5}$. This study controls the equilibrium condition rigorous with the inertial number maintained less than 10^{-6} .

4.2. Material behaviour

Figures 13–15 portray the stress-strain behaviour for the three specimens, respectively. Similar to the work of Frias Lopez et al. [30], the overfilled soil skeleton produces a lower peak and higher critical shear strength. Concerning the seepage impact, a slight content of eroded particles for the specimen FC60 induces a similar stress-strain response for the non-eroded and temporal eroded soil matrix. However, the high hydraulic gradient causes a great proportion of eroded particles for the specimen FC30, which reduces the peak shear strength of eroded soils compared with non-eroded soils. Regarding the local affected zone, the specimen FC40 induces a lower peak shear strength on the upstream side during the progressive seepage erosion. However, the long-term seepage impact reduces the peak shear strength on the downstream side for the specimen FC30. The results indicated that the weighted clustering coefficient is a better index

to determine the mechanical behaviour of the eroded soil matrix. A high magnitude of C_w corresponds to a higher peak shear strength.

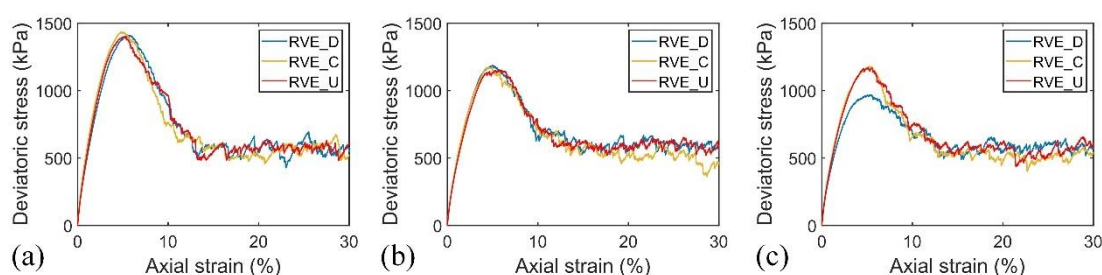


Figure 13. Stress-strain behaviour for the specimen FC30 at $t =$ (a) 0.0 s; (b) 0.1 s and (c) 0.2 s.

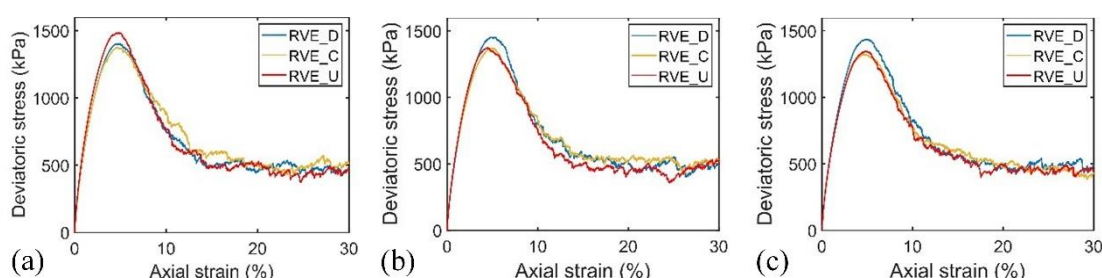


Figure 14. Stress-strain behaviour for the specimen FC40 at $t =$ (a) 0.0 s; (b) 0.1 s and (c) 0.2 s.

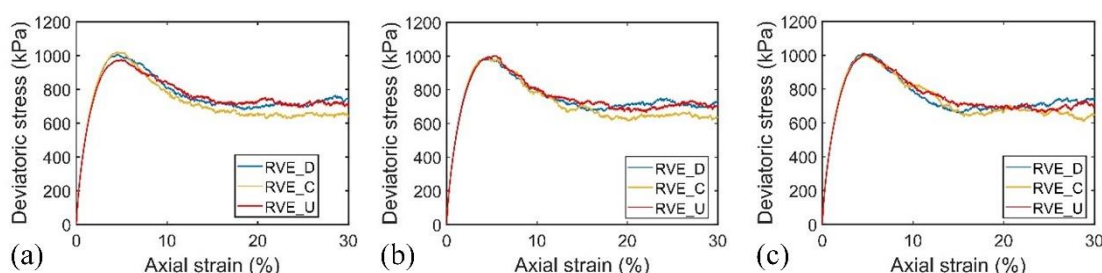


Figure 15. Stress-strain behaviour for the specimen FC60 at $t =$ (a) 0.0 s; (b) 0.1 s and (c) 0.2 s

5. Conclusions

This paper examines the influence of fine content on the onset and development of seepage failure using the coupled CFD-DEM software, mainly concerning the local affected zone in the hydraulic structure. Numerical simulations can capture the spatial and temporal changes of the structure to characterize the development of seepage erosion. The spatial distribution of particle velocity and drag force showed the soil matrix with a high content of fine particles would enhance the erosion resistance with a small displacement of particles. Three representative volume element boundaries were adopted to examine the impact of seepage duration on the characteristics of local geometry and the constitutive material behaviour. The rattlers and eroded particles were reduced as the erosion progressed. We adopted the simplest motifs in complex networks, the 3-cycles connections, to examine the local geometry of the internally unstable soil. The seepage failure started at the region adjacent to the retaining wall, which observes a sparse contact network. A fundamental quantity named as weighted clustering coefficient was proposed in this study to measure the density of the complex networks based on the information of the normal contact force. Results displayed that the specimen FC40 has the highest magnitude of the weighted clustering coefficient. The material behaviour indicated the weighted clustering coefficient is a better index to determine the mechani-

cal behaviour of the eroded soil matrix, with a higher magnitude corresponding to a higher peak shear strength. For the interactive-underfilled specimen, the high hydraulic gradient significantly reduced the shear strength, in particular for the downstream side. However, the seepage erosion reduced the peak shear strength on the upstream side for the interactive-overfilled specimen. The results indicated that the seepage erosion would reduce the stability of the structure at different positions for the sample prepared with various fine contents. It is necessary to examine a large-scale and long-term simulation in future work for a better understanding.

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