

This paper discusses a general nonlinear $SU(N)$ theory, intended to encompass both gravitational, gauge, vector, and axion fields on the same footing. The underlying basic element of this kind of theory is the Einstein-aether model, a subject studied earlier repeatedly by Balakin and collaborators. The action functional is given in eq. (2), where the Lagrangian parameter λ secures the normalization property of the Yang-Mills field. The Jacobson constitutive tensor [29] is augmented with two new coefficients, called C_5 and C_6 , because of the axionic generalization of the formalism. The axion potential $V(\phi)$ is chosen in the form (12), obeying the discrete symmetry property $\phi \rightarrow \phi + 2\pi n$. A main point in the present theory is that the decaying color aether transfers energy to the axion field. In the later parts of the paper, various solutions to the governing equations are discussed.

This is clearly a competent work, being an extension of earlier papers from the same milieu. One may say that the paper is to some extent speculative, but I think nevertheless that it ought to be of interest to the community. It should thus be publishable, practically in the form it is.

I have only one simple question: it is commonly accepted that the energy density of the axion field is a small but finite number; in physical units about 0.45 GeV/cm^3 . This should correspond to a finite and well-defined value of the axion wave amplitude. How can that comply with the axion field symmetry property $\phi \rightarrow \phi + 2\pi n$ advocated by the authors? Is observational cosmology restricted to the limiting case $\phi \rightarrow 0$?