

Review

Dynamically Coupled Reservoir and Wellbore Simulation Research in Two-Phase Flow Systems: A Critical Review

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Abstract

A coupled reservoir/wellbore simulator is essential for solving relevant flow phenomena with dynamic interactions between reservoir and wellbore in two-phase flow systems. A reservoir simulator or wellbore simulator alone fails to solve these transient flow phenomena. This paper summarises a critical review of the coupled reservoir and wellbore simulation. First, a wide application of the drift-flux (DF) models to simulate gas-liquid flow in a wellbore model coupling to a reservoir model is discussed. Then, the mechanisms of the coupled reservoir/wellbore simulator in two-phase flow systems are discussed, including the reservoir modeling, wellbore modeling, and their coupling schemes. Various examples of representative coupled simulators are presented. Finally, some case studies are reported to reveal dynamic interactions between wellbore and reservoir by using the coupled reservoir/wellbore simulators. This study gives the most up-to-date and systematic sights on the relevant research topic of reservoir/wellbore coupling, allowing for a better understanding of the coupled reservoir and wellbore simulation and its application in the oil and gas industry.

Keywords:

Reservoir model; Wellbore model; Coupling; Two-phase flow; Drift-flux model; Dynamic

flow phenomena

1. Introduction

The dynamic interaction between wellbore and reservoir research is essential to solve transient flow problems, such as wellbore storage, gas lift, coning, near-well cleanup, liquid loading, slugging, shut-in, start-up, smart wells, and reservoir management and forecast (Alberts et al., 2007; Da Silva and Jansen, 2015). In multi-phase (gas, water, and oil) flow, the pore saturation, and temperature and pressure propagation changes in a reservoir respond to propagation ranging timescales from hours to decades. Meanwhile, the wellbore's multi-phase flow, temperature, and pressure propagation have response times from seconds to hours. The dynamic dispatch between the wellbore and reservoir will lead to result in production instabilities, which can deteriorate the reservoir and wellbore production lifespan and ultimate recovery (Belfroid et al., 2005). Figure 1 provides a summary of common transient flow issues between reservoir and wellbore.

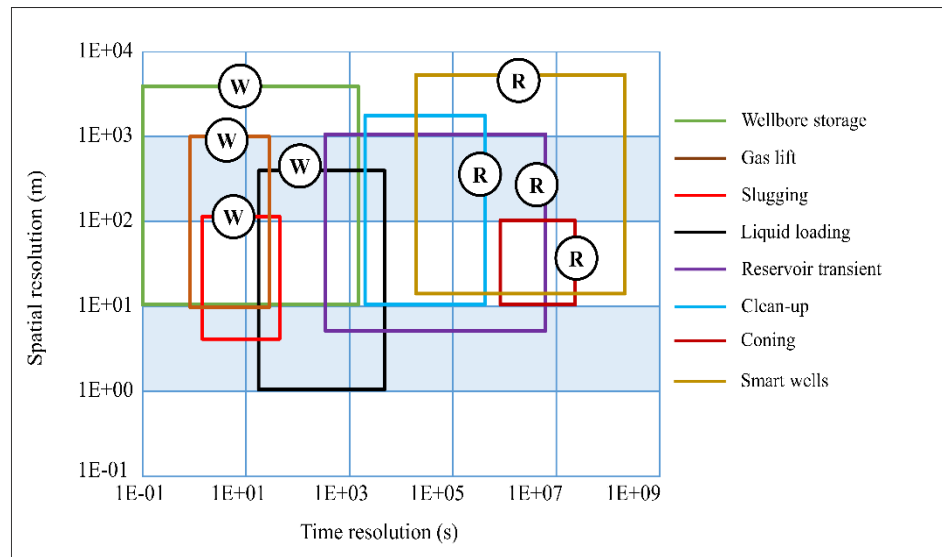


Figure 1. Spatial and time scales for the reservoir (R) and wellbore (W) phenomena. This figure was modified from Figure 1 in ref (Nennie et al., 2008).

Most wellbore simulators are employed to solve the flow issues in the wellbore area, whereas the reservoir simulators are used to simulate the reservoir phenomena. These

simulations apply either a dynamic reservoir model coupled with a steady wellbore model (Hasan et al., 1997; Hasan et al., 1998; Izgec et al., 2006; Kabir et al., 1996) or a dynamic wellbore model coupled with an analytical reservoir model (Coats et al., 2003; Kosmala et al., 2003; Zapata et al., 2001). Moreover, most commercial reservoir simulators (e.g., Eclipse and CMG) use steady-state values of bottom hole pressure (BHP) and tubing head pressure (THP). Similarly, most commercial wellbore flow simulators (e.g., [PIPESIM](#) and Prosper) assume a given multi-phase fluid flow rate from a reservoir into a wellbore for a reservoir pressure and constant BHP (Holmes et al., 1998). These commercial simulators do not square up a significant separation in spatial and time scales between reservoir and wellbore simulations. **These models disregard the dynamic interactions between the wellbore and reservoir, resulting in erroneous simulation predictions.**

In the past few years, some parties and scholars have realized the importance of the dynamic interactions between wellbore and reservoir and have been working on the development and application of wellbore-reservoir simulators. Millier (1980) might be the earliest scholar who studied transient interactions between wellbore and reservoir for well testing in geothermal wells (Miller, 1980). Other coupled reservoir/wellbore simulators were established to simulate dynamic flow problems between reservoirs and wellbores (Alberts et al., 2007; Almehaideb et al., 1989; Bahonar et al., 2011; Belfroid et al., 2005; Bhat et al., 2005; Cao et al., 2015; Holmes, 1983; Leemhuis et al., 2008; Livescu et al., 2010a; Nennie et al., 2008; Pan et al., 2014; Pourafshary, 2007; Sagen et al., 2011; Satti et al., 2019; Stone et al., 2002; Stone et al., 1989; Sturm et al., 2004; Tang et al., 2018; Twerda et al., 2011; Winterfeld, 1989; [Korzani et al., 2019](#); [Battistelli et al., 2020](#); [Basirat et al., 2020](#); [Galvao et al., 2020](#); [Raad et al., 2021](#); [Liao et al., 2021](#)). Numerical wellbore simulation contains a mass equation, momentum equation, and energy equation (Hasan et al., 2002; Shoham, 2006). Darcy's law is widely used to replace the classic momentum equation in reservoir simulation. In addition, most reservoir simulations suppose an iso-thermal condition, which implies that the energy equation is unnecessary (Aziz, 1979; Chen, 2007). **Coupled reservoir/wellbore simulations combine the**

coupling of the reservoir's partial differential equations (PDEs) and wellbore's PDEs. Three different coupling schemes (explicit, implicit, and fully implicit) are widely applied to construct dynamic reservoir/wellbore simulators (Iemcholvilert, 2013). The researchers mentioned above suggested their coupling models resolve specific flow problems between the reservoir and wellbore.

In this paper, we critically review numerical simulations of wellbore and reservoir models described in the open literature. While this review may not necessarily mention all the existing models, it has been ensured that a representative set of publications are covered. The prospect of this review paper is to help researchers quickly understand the mechanisms of reservoir/wellbore coupling and further improve its application.

We discuss the application of the drift-flux (DF) models for coupled reservoir-wellbore simulation in Section 2. Then, in Section 3, reservoir modeling, wellbore modeling, and their coupling schemes are described. Following that, a number of significant complexities of coupled reservoir/wellbore simulators are discussed in Section 4. Finally, Section 5 provides a summary of the findings.

2. The Drift-Flux Model

2.1 Role of drift-flux models

There are huge differences in the definitions of phase velocities between wellbore and reservoir simulations. As is often the case, the phase velocities in a reservoir are usually small, with negligible kinetic energy, for which Darcy's law is applicable to calculate the phase velocities (Chen, 2007). However, the kinetic energy must be considered in a wellbore simulation. The phase velocities are determined by the gas-liquid two-phase momentum equations (Hasan et al., 2002). The earliest model applied an empirical mixture equation to describe the two-phase flow (Beggs and Brill, 1973; Hagedorn and Brown, 1965; Moissis and Griffith, 1962; Orkiszewski, 1967; Ros, 1961). More complex models considered homogeneous flow, separated flow, and DF flow to simulate the two-phase momentum conservation. In the early 1980s, separate flow models were used for coupled numerical reservoir/wellbore

systems (Almehaideb et al., 1989; Stone et al., 1989). Holmes et al. (Holmes et al., 1998) outlined a multisegment well model to simulate gas-liquid flow in advanced wells. Two case studies of a horizontal well and a dual-lateral well were implemented to evaluate the performance of their multisegment well model. It was one of the first attempts to use a DF model to simulate the gas-liquid flow in a wellbore, rather than using traditional separated models (Holmes et al., 1998). As early as 1977, Holmes (Holmes, 1977) proposed his research to use the DF model to simulate flow rates of individual liquid or gas phases. After Holmes's publication, most of the subsequent researchers implemented DF models for coupled reservoir/wellbore research (Forouzanfar et al., 2015; Livescu et al., 2010a; Livescu et al., 2008; Pan et al., 2015; Pan et al., 2014; Pourafshary, 2007; Shirdel, 2013; Stone et al., 2002; Tang et al., 2017). Homogeneous (no-slip) models can also be used for coupled reservoir/wellbore simulations, but they fail to accurately calculate in-situ phase volume fractions and input volume fractions. Separate models need more computational requirements considering separate conservation equations for liquid and gas phases. Furthermore, the separate models will result in discontinuous flow rates or discontinuous derivatives of flow rates due to the existence of the discrete flow regimes (Da Silva and Jansen, 2015), which limits the application of these separate models for coupled reservoir/wellbore simulations. Conversely, DF models are considered accurate to predict gas volume fractions in gas-liquid flow (Woldesemayat and Ghajar, 2007). Tang et al. summarized two essential prerequisites for fully implicitly coupled reservoir/wellbore simulators (Tang et al., 2017). One is a mixture velocity for the gas or liquid phase, and the other is that the model must be continuous and differentiable. In general, a DF model is suitable for strongly coupling reservoir/wellbore simulators. More follow-up studies will apply DF models to simulate the gas-liquid flow in the wellbore coupling to the reservoir.

2.2 Description of a basic DF model

The DF models are widely applied in the nuclear industry for a while (Ishii and Mishima (Ishii et al., 1984), Bonizzi and Issa (Bonizzi and Issa, 2003), Hobiki and Ishii (Hibiki et al., 2004), Hoeld (Hoeld, 2007)). As indicated above, since Holmes (Holmes et al., 1998)

extensively applied a DF model for reservoir simulation, the use of DF models to couple the wellbore flow to a reservoir has been a common practice. The DF models consider the slip between fluid and gas phases and they are continuous and differentiable, making them suitable for coupling reservoir/wellbore simulations.

The basic DF model was first proposed by Zuber and Findlay (Zuber and Findlay, 1965). This one-dimensional DF model describes the slip effect between the gas and liquid phases. The general formulation is expressed in Eq. (1):

$$V_g = C_o V_m + V_d \quad (1)$$

where V_g is the flow velocity of the gas phase; C_o is a profile parameter, which reveals the effect of a non-uniform flow profile over a pipe cross-section; V_m is an average velocity of the mixture; V_d is the drift velocity considering the slip between the gas and liquid phases. V_m can be defined:

$$V_m = V_{sg} + V_{sl} = V_g \alpha_g + V_l (1 - \alpha_g) \quad (2)$$

where V_{sg} is the gas superficial velocity; V_{sl} is the liquid superficial velocity; V_l is the flow velocity of liquid; α_g is the in-situ gas volume fraction.

Figure 2 summarizes the mechanism of the basic DF model. The gas velocity is illustrated by a combination of the differences in gas concentration across the pipe section and the tendency of the gas phase to rise in the tubing owing to buoyancy (Livescu et al., 2010a; Shi et al., 2005a; Shi et al., 2005b).

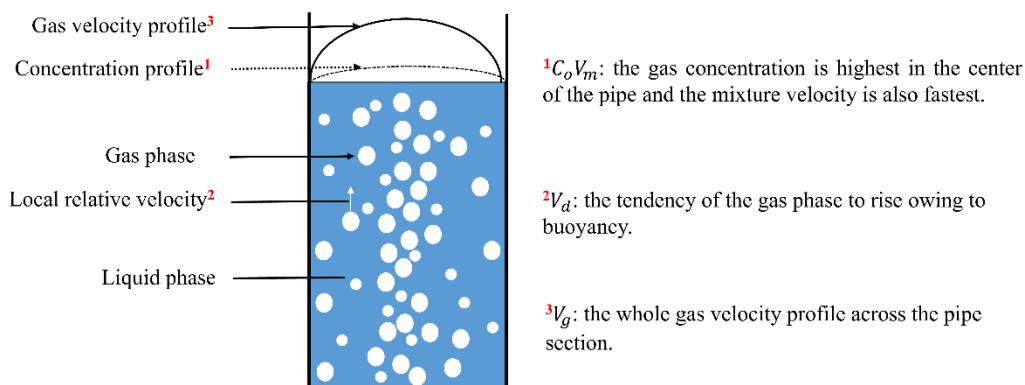


Figure 2. Schematic of DF models. This figure was modified from Figure 1 in ref (Shi et al., 2005a).

DF models require some empirical parameters. However, most of these parameters must be determined by experiments in vertical pipes with 2 inches or smaller diameters. These parameters can not be applicable in wellbore two-phase flow due to the huge differences in flow mechanisms between different large pipes and small pipes (Jepson and Taylor, 1993). Therefore, the overall goal is to form unified drift-flux models for coupled reservoir/wellbore simulators by adjusting these empirical parameters.

Since first published in 1965, the basic DF model has been refined by many researchers. In an early application in thermal-hydraulic analysis, Coddington and Macian (Coddington et al., 2002) presented a good review of 13 DF correlations to accurately forecast rod bundle void fractions in a nuclear reactor core. They summarized all the expressions of these correlations, showing that they can perform well for a broad range of experimental conditions (Bestion and design, 1990; Chexal et al., 1992; Gardner, 1980; Inoue et al., 1993; Ishii, 1977; Liao et al., 1985; Maier and Coddington, 1996; Pearson et al., 1984; Sonnenburg, 1989; Sun et al., 1980; Takeuchi et al., 1992; Zuber and Findlay, 1965). For the oil and gas industry, since Holmes (Holmes et al., 1998) published his work on an application of a drift-flux model in advanced wells, Hasan and Kabir (Ansari et al., 1994; Hasan et al., 1999; Hasan and Kabir, 1988a, b) and Ansari et al. (Ansari et al., 1994) have improved their DF correlations and these models are widely used for gas-liquid flow modeling in pipes (Flores et al., 1998; Hasan et al., 2010; Hasan et al., 2002; Petalas and Aziz, 2000). However, only a very limited number of DF models have been proven to be suitable for coupled reservoir/wellbore systems (Holmes, 1977; Shi et al., 2005a; Shi et al., 2005b; Tang et al., 2019a; Tang et al., 2019b).

2.3 Unified gas/liquid DF models for coupled reservoir/wellbore simulators

One of the most widely applicable DF models for coupled reservoir/wellbore simulations was proposed by Shi et al. (Shi et al., 2005a; Shi et al., 2005b). Their model could simulate concurrent and countercurrent flows, as well as complex advanced wells. They covered and

improved the DF model first presented by Holmes (Holmes, 1977). Oddie et al. (Oddie et al., 2003) performed large-scale flow experiments for multi-phase flows at various inclinations to determine these empirical parameters in Shi et al.'s model. Livescu et al. (Livescu et al., 2010a; Livescu et al., 2008; Livescu et al., 2010b) and Semenova et al. (Semenova et al., 2010) implemented this DF model, which was applied to Stanford's General Purpose Research Simulator, detailed in Cao (Cao, 2002) and Jiang (Jiang, 2007). Shi et al.'s model was also applied in other coupled reservoir/wellbore simulators and commercial simulators (Forouzanfar et al., 2015; Pan et al., 2015; Pan et al., 2014; Schlumberger, 2010). However, this model is suitable only for a pipe inclination from $+2^\circ$ (near horizontal) to $+90^\circ$ (vertical upward), which thereby limits its application. Figure 3 summarizes the use of multiplier $m(\theta)$ to compensate for gas/liquid deviation in Shi et al.'s model. It covered Hasan and Kabir's (Hasan et al., 1999) multiplier $m(\theta)$ and refined a valid range for θ from $[2^\circ, 30^\circ]$ to $[2^\circ, 90^\circ]$. A solution for the multiplier $m(\theta)$ for the gray region ($\theta \in [0^\circ, 2^\circ]$) has not been yet posed. As a DF model was developed from near-horizontal and vertical flow, it remained unsuited for horizontal or downward flow.

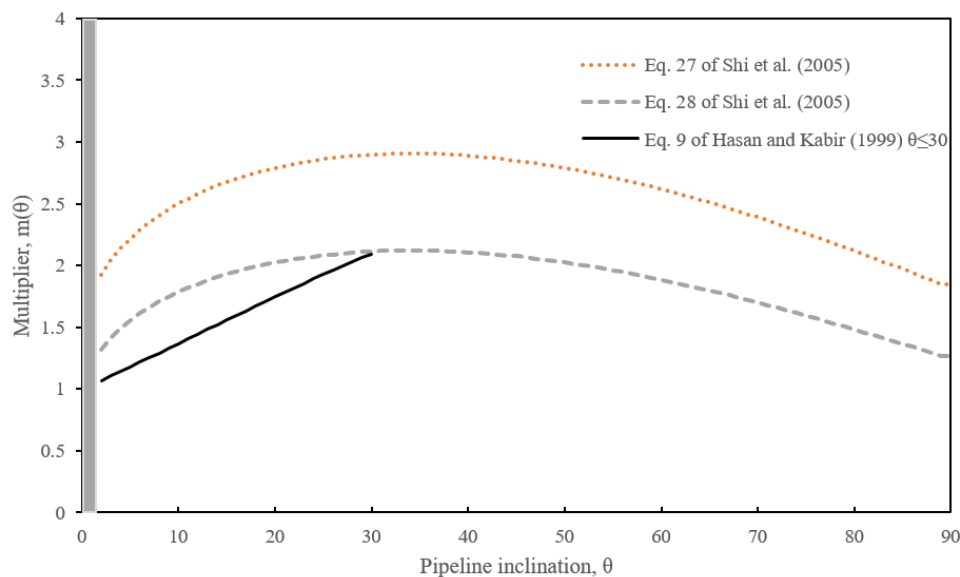


Figure 3. Three forms of the expression for inclination multiplier $m(\theta)$. This figure was modified from Figure 2 in ref (Tang et al., 2019b).

With an extensive application of horizontal wells and undulating horizontal wells, there

is a need to extend the inclination range of Shi et al.'s DF model (Luo et al., 2020; Piasecka et al., 2021; Wu et al., 2019). Li et al. (Li et al., 2011) presented a simple stratified equivalent model to extend Shi's model from -5° (downward flow) to $+90^\circ$, and their model is suitable for complex coupled systems. Choi et al. (Choi et al., 2012) raised an efficient DF model to forecast liquid holdups in a broad range of pipeline inclination angles. Choi et al., 2012's model was verified using an experimental dataset from Tulsa University Fluid Flow Projects (TUFPF) and a synthetic dataset from Oil and Gas simulator (OLGA) by the SPT group with inclination ($[-30^\circ, 90^\circ]$). A subsequent DF model proposed by Bhagwat and Ghajar (Bhagwat and Ghajar, 2014) represented the widest range of inclination ($\theta \in [-90^\circ, 90^\circ]$). The correlation of Bhagwat and Ghajar's model had a significant performance on the entire range of void fractions. However, all these DF models have never been applied in wellbore two-phase flow for coupling to a reservoir model. Other representative DF models suitable for coupled reservoir/wellbore simulators were developed by Tang et al. (Tang et al., 2019a; Tang et al., 2019b; Tang et al., 2017). First, Tang et al. (Tang et al., 2017) extended Shi et al.'s model to horizontal flow to simulate the liquid loading phenomenon in horizontal wells. Then, Tang et al. (Tang et al., 2019a; Tang et al., 2019b) developed a unified DF model for all wellbore inclinations ($\theta \in [-90^\circ, 90^\circ]$). Two datasets from TUFPF and OLGA-S were used to determine optimal parameters in Tang et al.'s model. Table 1 summarizes all the public sources from TUFPF (Choi, 2013), which can be worthy for subsequent new DF models. Tang et al. (Tang, 2019; Tang et al., 2019a; Tang et al., 2019b; Tang et al., 2017; Tang et al., 2018; Tang et al., 2019c) implemented their DF model into the General Unstructured Reservoir Utility (GURU) at Texas A&M University, detailed in Yan (Yan, 2017) and Olivares (Valbuena Olivares, 2015).

Table 1. Summary of datasets from TUFPF used for parameters optimization in Tang et al.'s model.

Time	Author	Inclination, θ (degrees)	Pipe diameter, D (cm)	Gas superficial velocity, V_g (m/s)	Liquid superficial velocity, V_L (m/s)
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1967	Eaton et al.	0	10.20	[0.112, 21.901]	[0.011, 2.108]
1972	Beggs	[-10, 10]	2.54	[0.299, 25.323]	[0.023, 5.203]
1976	Schmidt	90	5.08	[0.042, 13.146]	[0.070, 2.146]
1977	Cheremisinoff	0	6.35	[2.582, 25.241]	[0.017, 0.070]
1980	Mukherjee	[-90, 90]	3.81	[0.037, 41.310]	[0.015, 4.362]
1980	Akpan	0	7.62	[0.199, 5.458]	[0.137, 1.701]
1982	Vongvuthipornchai	0	7.62	[0.061, 2.938]	[0.070, 2.146]
1983	Minami	0	7.79	[0.475, 16.590]	[0.005, 0.951]
1986	Caetano	90	6.34	[0.023, 22.859]	[0.002, 3.579]
1986	Kouba	0	7.62	[0.302, 7.361]	[0.152, 2.137]
1986	Rothe et al.	[-2, 0]	17.10	[0.610, 4.633]	[0.061, 1.830]
1992	Felizola	[0, 90]	5.10	[0.390, 3.360]	[0.050, 1.490]
1996	Roumazeilles et al.	[-30, 0]	5.10	[0.914, 9.357]	[0.884, 2.438]
1996	Brill et al.	[-10, 10]	7.79	[3.629, 12.656]	[0.004, 0.046]
1999	Meng	[-2, 2]	5.08	[4.600, 26.600]	[0.001, 0.054]
2000	Abdul-Majeed	0	5.08	[0.196, 49.908]	[0.002, 1.825]
2005	Fan	[-2, 2]	5.08	[4.930, 25.700]	[0.0003, 0.052]
2005	Johnson	[0, 5]	10.00	[0.711, 4.523]	[0.019, 0.605]
2011	Yuan	[30, 90]	7.62	[9.900, 36.000]	[0.005, 0.100]
2012	Magrini et al.	[0, 90]	7.62	[36.630, 82.320]	[0.003, 0.040]
2012	Guner	[0, 45]	7.62	[1.485, 39.388]	[0.010, 0.100]
2013	Alsaadi	[2, 30]	7.62	[1.829, 39.992]	[0.010, 0.101]

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218 We reviewed several DF models for simulating gas-liquid flow in the wellbore at various
 219 inclinations. Shi et al.'s model was regarded as the DF model with the widest application for
 220 coupled reservoir/wellbore simulators. Tang et al. claimed that their model was unified and
 221 suitable for any coupled systems, whereas other DF models have the potential to be applied
 222 in coupled reservoir/wellbore simulators in future work. Table 2 presents the correlations of
 223 those advanced DF models discussed above.

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225

Table 2. Several unified DF models for coupled reservoir/wellbore simulation.

Author	Inclination θ (degrees)	Correlation
Shi et al. (2005)	[+2, +90]	$C_o = \frac{A}{1+(A-1)\gamma^2}$ $V_d = \frac{(1-\alpha_g C_o) C_o K(\alpha_g) V_c}{\alpha_g C_o \sqrt{\frac{\rho_g}{\rho_l} + 1 - \alpha_g C_o}}$
Li et al. (2011)	[-5, +90]	$\alpha_g = \frac{V_g \alpha_g}{C_o(\alpha_g, X_p) V_m + V_d(\alpha_g, X_p)}$ $\alpha_g(\theta) = C_1(\theta) (\alpha_g)^{SE} + [1 - C_1(\theta)] (\alpha_g)^{DF}$ $C_1(\theta) = \begin{cases} 1 & \text{for } \theta \in [\theta^{SE}, 0^\circ] \\ \exp\left(-\beta \frac{\theta^2}{(\theta^{DF})^2 - \theta^2}\right) & \text{for } \theta \in [0^\circ, \theta^{DF}] \\ 0 & \text{for } \theta \in [\theta^{DF}, 90^\circ] \end{cases}$
Choi et al. (2012)	[-30, 90]	$C_o = \frac{2}{1+(Re/1000)^2} + \frac{1.2-0.2 \sqrt{\frac{\rho_g}{\rho_l}} (1-\exp(-18\alpha_g))}{1+(1000/Re)^2}$ $V_d = A \cos \theta + B \left(\frac{g \sigma \Delta \rho}{\rho_l^2} \right)^{1/4} \sin \theta$
Bhagwat and Ghajar (2014)	[-90, 90]	$C_o = \frac{2-(\rho_g/\rho_l)^2}{1+(Re/1000)^2} + \frac{[(1+(1-\alpha)^{2/5} \sqrt{(1+(\frac{\rho_g}{\rho_l})^2 \cos \theta)) / (1+\cos \theta)})]^{(1-\alpha)^{2/5}}}{1+(1000/Re)^2} + C_{0.1}$ $V_d = (0.35 \sin \theta + 0.45 \cos \theta) \times \sqrt{\frac{g D (\rho_l - \rho_g)}{\rho_l}} (1-\alpha_g) C_2 C_3 C_4$ $C_{0.1} = \begin{cases} 0, & -50^\circ \leq \theta \leq 0^\circ \text{ and } Fr_{sg} \leq 0.1 \\ (C_1 - C_1 \sqrt{\frac{\rho_g}{\rho_l}}) [(2.6 - \beta)^{0.15} - \sqrt{f_{tp}}] (1-x)^{1.5}, & \text{otherwise} \end{cases}$ $C_2 = \begin{cases} \left[\frac{0.434}{\log(\mu_l/0.001)} \right]^{0.15}, & (\mu_l/0.001) > 10 \\ 1, & (\mu_l/0.001) \leq 10 \end{cases}$ $C_3 = \begin{cases} (La/0.025)^{0.9}, & La > 0.025 \\ 1, & La \leq 0.025 \end{cases}$ $C_4 = \begin{cases} -1, & -50^\circ \leq \theta \leq 0^\circ \text{ and } Fr_{sg} \leq 0.1 \\ 1, & \text{otherwise} \end{cases}$

Tang et al.
(2017)

[0, 90]

$$C_o = \frac{A}{1+(A-1)\gamma^2}$$

$$V_d = \frac{(1-\alpha_g C_o) C_o m V_c}{\alpha_g C_o \sqrt{\frac{\rho_g}{\rho_l} + 1 - \alpha_g C_o}}, \quad 0^\circ \leq \theta \leq 2^\circ$$

$$V_c = \sqrt{\frac{g D (\rho_l - \rho_g)}{\rho_l}}$$

$$V_d = \frac{(1-\alpha_g C_o) C_o K(\alpha_g) V_c}{\alpha_g C_o \sqrt{\frac{\rho_g}{\rho_l} + 1 - \alpha_g C_o}}, \quad 2^\circ \leq \theta \leq 90^\circ$$

$$V_{dm} = V_d \times m \times f(\theta)$$

Tang et al.
(2019)

[-90, 90]

$$C_o = \frac{A}{1+(A-1) \times \min \left\{ \left[\frac{\max \left(\alpha_g, \frac{\alpha_g |V_m|}{V_{sgf}} \right) - B}{1-B} \right]^2, 1 \right\}}$$

$$V_d = (m_1 V_d^v \sin \theta + \left\{ 1 - \frac{2}{1 + \exp[50 \times \sin(\theta + m_2 V_m)]} \right\} V_d^h \cos \theta) \times$$

$$\left(1 + \frac{1000}{Re_1 + 1000} \right)^{m^3}$$

227 where C_o is the profile parameter; V_d is the drift velocity; A is the value of the profile
 228 parameter in the bubble and slug flow regimes; γ is the term involving to cause C_o to reduce
 229 to 1.0; α_g is the in-situ gas volume fraction; $K(\alpha_g)$ is the critical Kutateladze number; V_c is the
 230 characteristic velocity. X_p is the DF model parameter vector; $C_1(\theta)$ is the tunable weight
 231 function; V_g is the in-situ gas velocity; V_m is the mixture velocity. θ is the pipe inclination
 232 angle; Re is the Reynolds number; σ is the surface tension; A, B is the coefficient constant.
 233 Fr_{sg} is the Froude number; $C_{0,1}, C_2, C_3, C_4$ is the coefficient constant; f_{tp} is the friction factor;
 234 μ_l is the liquid viscosity; La is the Laplace variable. m is the coefficient constant; $f(\theta)$
 235 represents the influence of pipe inclination; V_{dm} is the modified V_d . V_{sgf} is the gas flooding
 236 velocity; m_1, m_2, m_3 is the coefficient constant; V_d^v is the vertical drift velocity; V_d^h is the
 237 horizontal drift velocity.

3. Coupled reservoir/wellbore modeling

3.1 Reservoir modeling

The numerical simulation of reservoir flow is based on a series of conservation equations, where typical variables are pressure and phase saturations (Chen et al., 2006). Darcy's law is selected to describe fluid velocities through porous media in the reservoirs. Hydrocarbons in a reservoir are described by using black oil or compositional models. Other auxiliary equations involve interfacial tensions and capillary effects. For example, we consider three-phase Darcy flow in the porous media in compositional system. Eqs. (3) and (4) express the mass conservation equations for hydrocarbon components and the water phase (Livescu et al., 2010a; Livescu et al., 2008; Tang, 2019; Tang et al., 2019c). Usually, hydrocarbon components are not present in the water phase. Eqs. (5) is the total energy balance equation.

$$\nabla \left[kA \left(\frac{k_{ro}}{\mu_o} x_c \tilde{\rho}_o \nabla (P_o - \rho_o g z) + \frac{k_{rg}}{\mu_g} y_c \tilde{\rho}_g \nabla (P_g - \rho_g g z) \right) \right] = V_b \frac{\partial}{\partial t} \left(\phi (S_o x_c \tilde{\rho}_o + S_g y_c \tilde{\rho}_g) \right) - n_{c,s} \quad (3)$$

$$\nabla \left[kA \left(\frac{k_{rw}}{\mu_w} \tilde{\rho}_w \nabla (P_w - \rho_w g z) \right) \right] = V_b \frac{\partial}{\partial t} \left(\phi (S_w \tilde{\rho}_w) \right) - n_{w,s} \quad (4)$$

$$\nabla \left[kA \sum_{\alpha} H_{\alpha} \frac{k_{r\alpha}}{\mu_{\alpha}} \tilde{\rho}_{\alpha} \nabla (P_{\alpha} - \rho_{\alpha} g z) \right] = V_b \frac{\partial}{\partial t} \left(\phi \sum_{\alpha} S_{\alpha} \tilde{\rho}_{\alpha} U_{\alpha} \right) + Q_{loss} + q_H \quad (5)$$

where k is the absolute permeability; V_b is the bulk rock volume; k_{α} ($\alpha=o, g, w$) is the phase relative permeability; μ_{α} ($\alpha=o, g, w$) is the phase viscosity; A is a cross-sectional area; P_{α} ($\alpha=o, g, w$) is phase pressure; $\tilde{\rho}_{\alpha}$ ($\alpha=o, g, w$) is the molecular density; ρ_{α} ($\alpha=o, g, w$) is the mass density; g is the gravitational acceleration; z is the depth; ϕ is the porosity; x_c and y_c are the mole fractions of hydrocarbon component c in the oil and gas phases, respectively; S_{α} ($\alpha=o, g, w$) is a phase saturation; $n_{c,s}$ is a sink term; $n_{w,s}$ is a source term; H_{α} ($\alpha=o, g, w$) is the enthalpy of the phase; U_{α} ($\alpha=o, g, w$) is the specific internal energy of the phase; Q_{loss} represents the heat exchange between the reservoir and wellbore; q_H is the internal source terms for the total energy.

These partial differential equations (PDEs) are usually resolved by a fully implicit time discretization and a finite-volume space discretization, although sometimes a finite element discretization is used in space (Da Silva and Jansen, 2015; Valbuena Olivares, 2015; Yoshida,

2016). An adaptive time-stepping scheme can also be applied for reservoir simulation. In a compositional simulation, the fluid properties of gas and oil phases are determined by vapor-liquid flash calculations based on an equation of state (EOS).

3.2 Wellbore modeling

Numerical simulation of wellbore flow is based on a set of mass, momentum, and energy conservation equations. The mass conservations for hydrocarbons and water in wellbore flow are shown in Eqs. (5) and (6) (Livescu et al., 2010a; Livescu et al., 2008; Tang, 2019; Tang et al., 2019c). The primary variables consist of hydrocarbon component molar fractions, pressure, volumetric holdups, and a mixture velocity. The superficial velocity phases in Eqs. (6) and (7) are determined by a DF model discussed in Section 2. Considering the case of two-phase flow, a momentum conservation equation for a pressure drop is obtained (Ishii and Hibiki, 2010). Eq. (8) represents the total pressure loss for the whole wellbore (Govier and Aziz, 1972; Hasan et al., 2010; Hasan et al., 2002; Livescu et al., 2010a). The pressure drop equation consists of the pressure losses due to gravity, acceleration, and frictional effects. Eq. (9) represents the total energy balance for the whole wellbore system. In thermal reservoir/wellbore simulators, the energy conservation equation is used to calculate temperature (Bahonar et al., 2011; Dong et al., 2011; Hasan et al., 2002; Ramey, 1962; Satter, 1965; Xiong et al., 2015). Also, the phase-equilibrium equations are chosen to calculate each phase's mole fractions and composition under specific pressure and temperature conditions.

$$\nabla \left[A(x_c \tilde{\rho}_o v_{so} + y_c \tilde{\rho}_g v_{sg}) \right] = V_{\text{seg}} \frac{\partial}{\partial t} (\alpha_o x_c \tilde{\rho}_o + \alpha_g y_c \tilde{\rho}_g) + n_{c,s} \quad (6)$$

$$\nabla [A \tilde{\rho}_w v_{sw}] = V_{\text{seg}} \frac{\partial}{\partial t} (\alpha_w \tilde{\rho}_w) + n_{w,s} \quad (7)$$

$$\frac{\partial P}{\partial z} + \frac{\partial \rho_m v_m^2}{\partial z} + \frac{f_{tp} \rho_m v_m |v_m|}{2d} + \rho_m g \cos \theta = 0 \quad (8)$$

$$\frac{\partial}{\partial z} \sum_f \tilde{\rho}_f \alpha_f (U_f + \frac{1}{2} V_f^2) + \frac{\partial}{\partial z} \sum_f \tilde{\rho}_f V_{sf} (H_f + \frac{1}{2} V_f^2) = \sum_f \tilde{\rho}_f V_{sf} g \cos \theta + Q_{\text{loss}} + q_H \quad (9)$$

where $\alpha_f (f=o, g, w)$ is the volumetric holdup; $v_f (f=o, g, w)$ is the superficial velocity; ρ_m is the mixture density; v_m is the average mixture velocity; d is the pipe inner diameter; f_{tp} is the friction factor; $H_f (f=o, g, w)$ is the phase enthalpy; $U_f (f=o, g, w)$ is the phase specific internal

energy.

These PDEs of mass and momentum conservation equations can be one-dimensional, usually discretized in space by the finite-volume methods along the axial direction of a wellbore. Usually, a staggered grid arrangement is adopted for the discretization in space, which means that the variables of pressure, temperature, and holdup are calculated at the centre of each segment while the velocities are solved at the face of each segment (Livescu et al., 2010a; Tang et al., 2019c; Yoshida, 2016).

3.3 Numerical implementation of coupling reservoir and wellbore models

Implementing of the reservoir and wellbore models numerically requires the coupling of two sets of PDEs. The reservoir's PDEs and wellbore's PDEs are coupled through source and sink terms in mass balance equations. Meanwhile, the reservoir pressure at the reservoir/wellbore surface must match the wellbore segment pressure. The reservoir models and wellbore models are concurrently solved at each time step. Important control parameters (e.g. phase flow rate, tubing head, and bottom hole pressure) are transferred between these two sets of PDEs. The primary reservoir variables include the moles of hydrocarbon components and water, pressure, temperature, and saturation. The primary wellbore variables contain the moles of hydrocarbon components and water, pressure, temperature, volumetric holdup, and mixture velocity. Once the primary variables in the reservoir and wellbore are resolved, the other variables will be updated in the whole coupled systems.

There are three different types of coupling schemes used in a coupled reservoir/wellbore simulation: explicit, implicit, and fully implicit (Bahonar et al., 2011; Iemcholvilert, 2013; Stone et al., 1989; Tang, 2019).

In an explicit coupling scheme, a reservoir and a wellbore are treated as different domains and resolved at different time steps (Gao, 2014; Redick, 2017). The explicit coupling scheme will balance the wellbore and reservoir at the same time step level. This scheme produces less computational burden and is highly flexible for a coupled system. A case in point is an explicit calculation of bottom hole pressure (*BHP*) as a controlling parameter.

Wellbore models will calculate the BHP at a specific time step n for a phase rate Q given by the reservoir model and THP . Then one uses the BHP as a well boundary datum to solve reservoir models to obtain a new phase rate Q_{new} . This new phase rate Q_{new} is then selected as a new input for wellbore simulation at the next synchronization time step $n+1$. Figure 4 summarizes the schematic of the explicit coupling scheme discussed above.

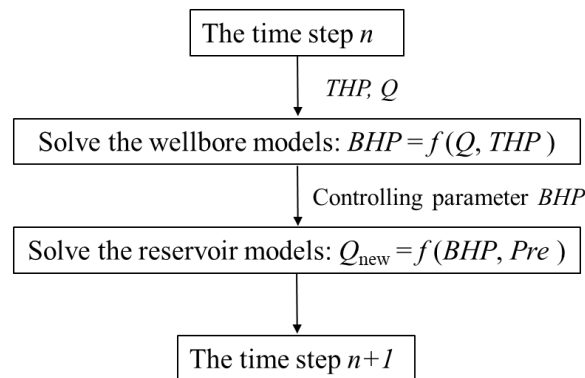


Figure 4. Schematic of the explicit coupling scheme.

In an implicit coupling scheme, a reservoir and a wellbore are also treated as different domains but balanced at a Newton iteration level. This scheme demands more computational effort than the previous explicit coupling scheme due to the iterative calculation associated with the time step level (Gao, 2014; Trick, 1998). In the implicit case, a requirement for coupling actions must be determined at the beginning of each Newton iteration. The controlling parameter BHP calculated by wellbore models will pass to the reservoir models as a boundary constraint. In the same time step, one uses this converged BHP to solve reservoir models and check the convergence of the reservoir equations. If they converge, one proceeds to the next time step. If they diverge, the iterations are repeated to update Q and THP to calculate the reservoir model and wellbore model until the Newton solution of coupled processes converges. The schematic of the implicit coupling scheme is shown in Figure 5.

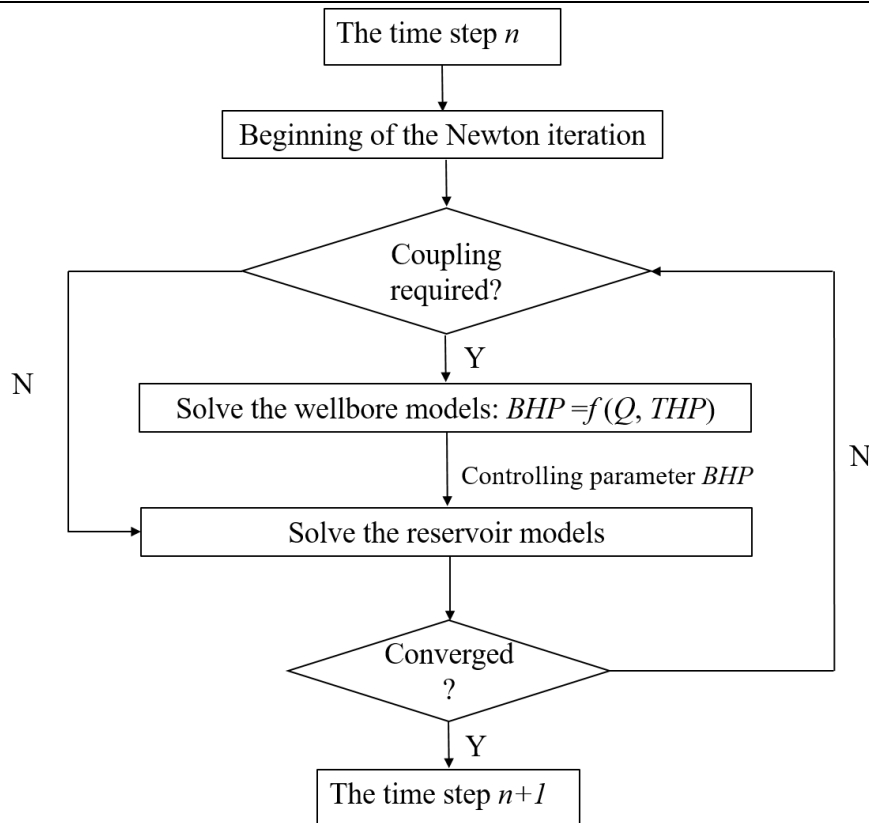


Figure 5. Schematic of the implicit coupling scheme.

In a fully implicit coupling scheme, a reservoir and a wellbore are treated as one domain. Reservoir models and wellbore models are solved simultaneously at each Newton iteration level (Gao, 2014; Ghorayeb et al., 2003). The Jacobian matrix in the fully implicit scheme is significantly larger than that for independent wellbore or reservoir simulators. The fully implicit coupling scheme needs the most computational effort and produces the most stable solution for a coupled reservoir/wellbore simulation among these three schemes. For a fully implicit case, synchronization points need to be defined where information from the reservoir is synchronously passed to the wellbore. The primary variable parameters (i.e. BHP, reservoir pressure, saturation, moles of hydrocarbon component, and holdups) are solved simultaneously. The fully implicit scheme will check whether the Newton iteration is converged or not. If yes, one proceeds to the next time step. Otherwise, one repeats iteration loops until a Newton iteration converged. The schematic of the fully implicit coupling scheme is presented in Figure 6.

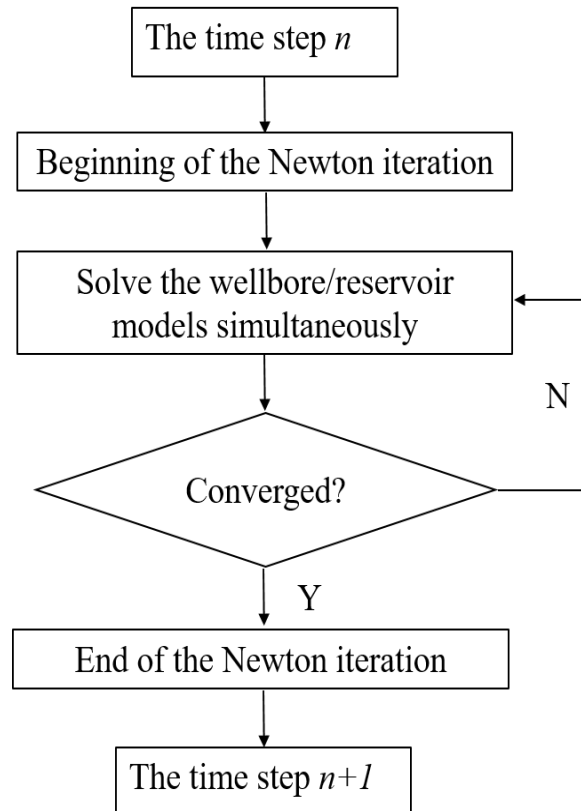


Figure 6. Schematic of the fully implicit coupling scheme.

In the above discussed three coupling schemes, the explicit coupling method only balances the reservoir model and wellbore model at the beginning of each time step and fails to generate an accurate reservoir state or wellbore state at the end of each time step. The implicit coupling scheme achieves equilibrium between the reservoir model and wellbore model at Newton iteration level. However, the implicit solution's correctness relies on the coupling frequency, and the time step convergence is simply based on the reservoir model. The fully implicit scheme is regarded as the most stable and most accurate coupling method due to the reservoir model and wellbore model being solved simultaneously at Newton iteration level.

We review the most representative examples of coupled reservoir/wellbore simulations in open publications. Table 3 summarizes the most relevant aspects of various coupling examples.

Table 3. Overview of the most representative examples of coupled reservoir/wellbore simulators.

Author (s)	Coupled reservoir/wellbore simulation	Coupling scheme
Stone et al. (1989)	M, T, B, SF, H	F
Almehaideb et al. (1989)	M, N, B, SF, V	F
Winterfeld (1989)	M, N, B, SM, V	F
Holmes et al. (1998)	M, ?, B, DF, M	F
Stone et al. (2002)	M, T, C, DF, M	I
Coats et al. (2004)	M, N, C, ?, M	F
Bhat et al. (2005)	S, T, B, ?, H	I
Nennie et al. (2007)	M, T, B, SF, M	E
Sagen et al. (2007)	M, T, B, ?, M	F
Pourafshary (2007)	M, T, C, DF, V	F
Leemhuis et al. (2008)	M, T, B, SF, M	E
Livescu et al. (2009)	M, T, C, DF, M	F
Livescu et al. (2010)	M, T, B, DF, M	F
Semenova et al. (2010)	M, T, C, DF, M	F
Twerda et al. (2011)	M, T, B, SF, M	F
Bahonar et al. (2011)	S, T, B, ?, V	F
Shirdel (2013)	S, N, C, ?, H	F
Pan and Oldenburg (2014)	M, T, C, DF, V	F
Gao (2014)	M, N, B, SM, V&H	I
Forouzanfar et al. (2015)	M, T, C, DF, V	F
Cao et al (2015)	M, N, C, ?, M	F
Olivares (2015)	M, N, C, SM, V	F
Redick (2017)	M, N, B, ?, V	I
Tang et al. (2018)	M, N, C, DF, H	F
Galvao et al. (2019)	S, T, C, ?, V	E
Galvao et al. (2020)	S, T, C, ?, V	E
Basirat et al. (2020)	M, T, C, DF, V	F
Battistelli et al. (2020)	M, T, C, DF, V	F
Raad et al. (2021)	M, T, B, ?, V	E
Liao et al. (2021)	M, T, B, ?, H	F

Legend:

Coupled reservoir/wellbore simulation: M=multiphase; S=singlephase; T=thermal;
N=non-thermal; B=black oil; C=compositional; SF-separated flow; SM-semi empirical model;
DF-Drift-Flux; V-vertical well; H-horizontal well; M-multilateral well; ?-unknown.

Coupling scheme: F-fully implicit; I-implicit; E-explicit.

4. Applications of coupled reservoir/wellbore simulators

4.1 Typical application 1- gas coning

Gas coning is a common gas flow phenomenon where the reservoir's gas-oil contact migrates slowly to the wellbore due to the drawdown of the oil production in the horizontal or deviated wells. The progress of the gas cone is a gradual transitional process. Once the gas-oil contact reaches the wellbore, the gas-liquid mixture in the wellbore abruptly alters with a steep increase in the gas-oil ratio (GOR), resulting in a substantial decrease in bottom hole pressure. The reservoir simulation usually applies the lift curve to simulate gas coning, while varied GOR values will result in various bottom hole pressures. The variation of bottom hole pressure can reflect the occurrence of gas coning. However, reservoir simulation lacks effective simulation of the dynamic transient in the wellbore. Single reservoir simulation can hardly describe the gas breakthrough phenomenon due to a constant productivity index applied. A dynamic coupled wellbore-reservoir simulator was developed to simulate gas coning problems by Nennie et al. and Leemhuis et al. (Belfroid et al., 2005; Leemhuis et al., 2008; Nennie et al., 2008). Figure 4 shows the BHP at different inflow control valves (ICVs) versus time during the gas breakthrough in a horizontal well with multizone completion by the coupled reservoir/wellbore simulator and simulator MoReS. MoRes is a reservoir simulator that can handle non-fractured and fractured reservoirs. It can be drawn from Figure 4 that the simulation points between the coupled reservoir/wellbore simulator and simulator MoReS are slightly different. A smaller BHP in the coupled reservoir/wellbore simulator makes a larger mass flow to the wellbore than the simulator MoRes, which results in the gas breakthrough occurring 15 hours earlier in the coupled simulator. Moreover, the pressure

development after gas coning between the coupled reservoir/wellbore simulator and the simulator MoRes indicates a significant difference. The pressure derivative performs much steeper after the gas breakthrough than before. This case reveals how a dynamic reservoir/wellbore simulator influences the gas coning phenomenon more accurately than a single reservoir simulator.

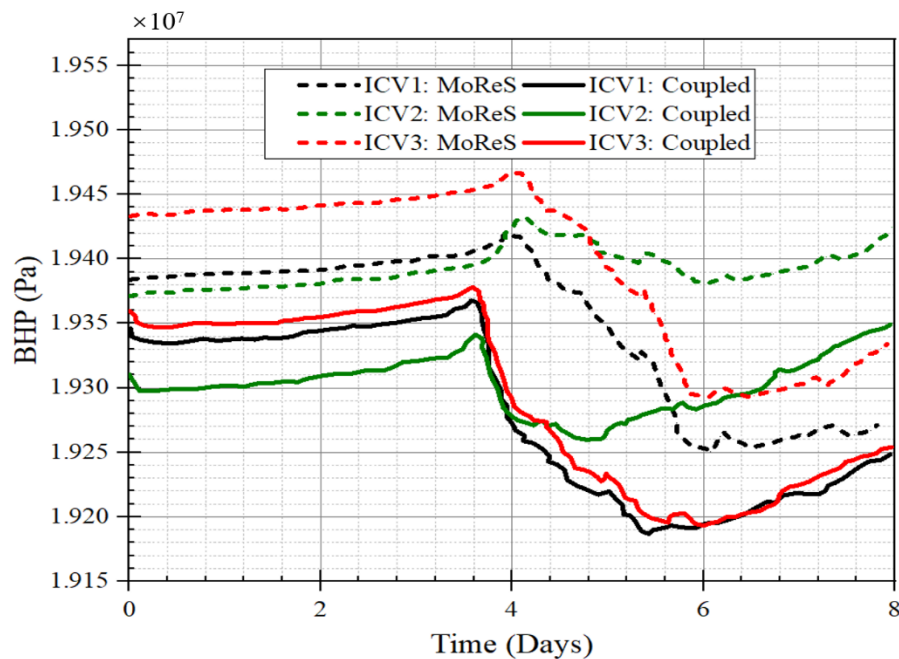


Figure 7. BHP at different ICVs versus time during gas breakthrough. This figure was modified from Figure 7 in ref (Nennie et al., 2008).

4.2 Typical application 2-well storage effect

Well storage effect is usually referred to as the well dynamics during start-up and shut-in. Both start-up and shut-in will imply a strong interaction between the wellbore and the reservoir. During the well testing, the pressure measurements are collected within the wellbore instead of the reservoir. The pressure recorder is employed to measure the reservoir responses, unfortunately, the wellbore responses are also gathered. Moreover, the wellbore responses are much faster than the reservoir responses. Consequently, the wellbore storage effect between the wellbore and reservoir must be considered during well testing. The researchers from Integrated System Approach Petroleum Production (ISAPP) simulated the dynamic well storage phenomenon to reveal the necessity of a coupled reservoir/wellbore

simulator (Alberts et al., 2007; Nennie et al., 2008). They stated that the coupled reservoir/wellbore model was a Matlab coupling between a single reservoir simulator MoRes and a single wellbore simulator OLGA. Figure 8 presents the BHP versus time for simulations during shut-in and start-up periods from the coupled simulator and OLGA. OLGA is a multiphase flow simulator that can simulate transient flow behavior in the wellbore. According to Figure 8 (a), the BHP simulated by OLGA soon reached the original reservoir pressure within several minutes while the coupled simulator reached the reservoir pressure after six hours during the build-up period after the shut-in operation. Moreover, the reservoir pressure of the coupled simulator was smaller than the initial reservoir pressure owing to reservoir depletion. The wellbore storage effect also influences the start-up period, referred to Figure 8 (b). The OLGA simulator can simulate the well storage but overrate the BHP due to the missing reservoir depletion. Therefore, the interactions between wellbore and reservoir should be taken into consideration for dynamic procedures such as shut-in, start-up, depletion, and clean-up.

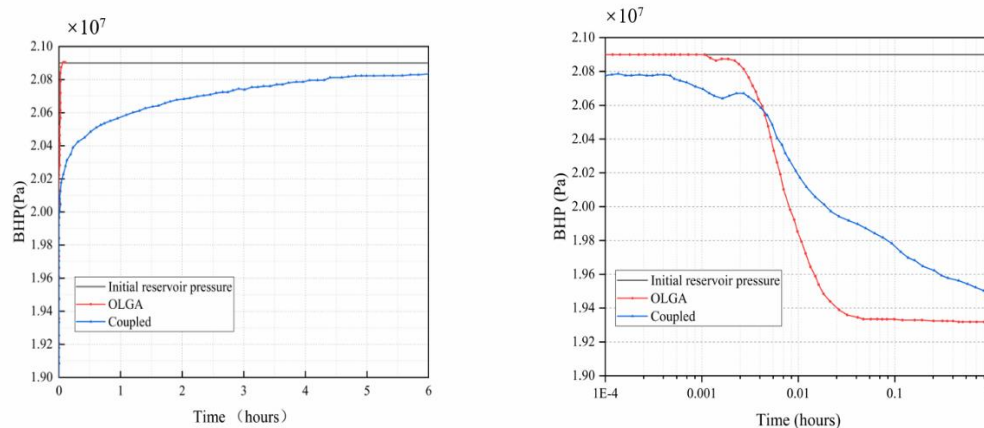


Figure 8. Comparisons of BHP versus time (a) build-up period after shut-in and (b) start-up period from coupled simulator and OLGA. This figure was modified from Figures 9 and 14 in ref (Nennie et al., 2008).

4.3 Typical application 3- liquid loading

Liquid loading is a serious issue when the gas rate fails to lift all the liquids to the surface in most mature gas fields. An accumulation of unproduced liquid in a wellbore causes

sudden decreases in gas production and drastic fluctuations in wellbore pressure (Lea and Nickens, 2004). Liquid loading can hardly be comprehensively simulated by a reservoir simulator or wellbore simulator due to the flow transience between reservoir and wellbore. Tang et al. (Tang, 2019) developed a fully implicitly coupled reservoir/wellbore simulator to describe the liquid loading phenomenon in horizontal wells. Figure 9 compares the gas-producing rate and the pressure of a perforated well segment from Tang et al.'s coupled simulator and Eclipse 300 (E300). A relevant DF model used in E300 was detailed by Shi et al. (Shi et al., 2005b). The simulation results revealed a sharp drop in the gas rate and a steep increase in the pressure at the perforated segment at approximately the 60th day. Tang et al.'s simulator forecasted the onset of the liquid loading phenomenon around one day earlier than E300.

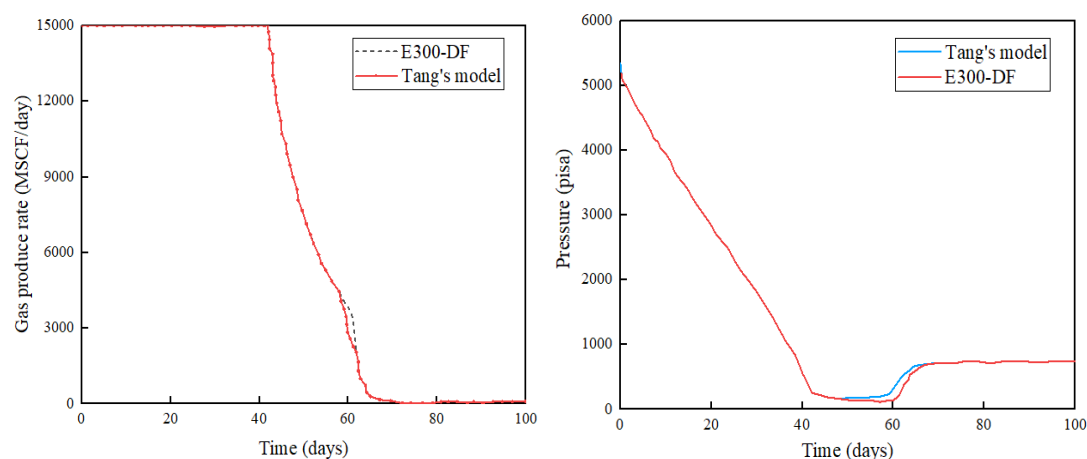


Figure 9. Comparisons of (a) gas-producing rate and (b) the pressure of a perforated segment from Tang et al.'s coupled reservoir/wellbore simulator and E300. This figure was modified from Figures 4.7 and 86 in ref (Tang, 2019).

4.4 Typical application 4- general scenarios

A coupled reservoir/wellbore simulator containing black oil and compositional models proposed by Livescu et al. (Livescu et al., 2010a; Livescu et al., 2008) was used in some complex cases. Their case studies revealed various applications for simulating three-phase steady-state flow, gas condensate, thermal flow, phase changes, and compositional fluctuations. Various wellbore types including vertical wells, horizontal wells, and

multilateral wells are connected to the reservoir by different completion methods. As illustrated in Figure 10, the whole wellbore is split into multiple segments coupling the reservoir grids. The variable parameters in the coupled reservoir/wellbore systems include pressure, temperature, mole fractions, and in-situ phase fractions. All these variable parameters are captured at different time steps to accurately describe physical phenomena both in a wellbore and a reservoir. The underlying models are flexible to solve advanced completions and complex downhole networks. Livescu et al.'s coupled simulator has been proven with broad applicability and robustness.

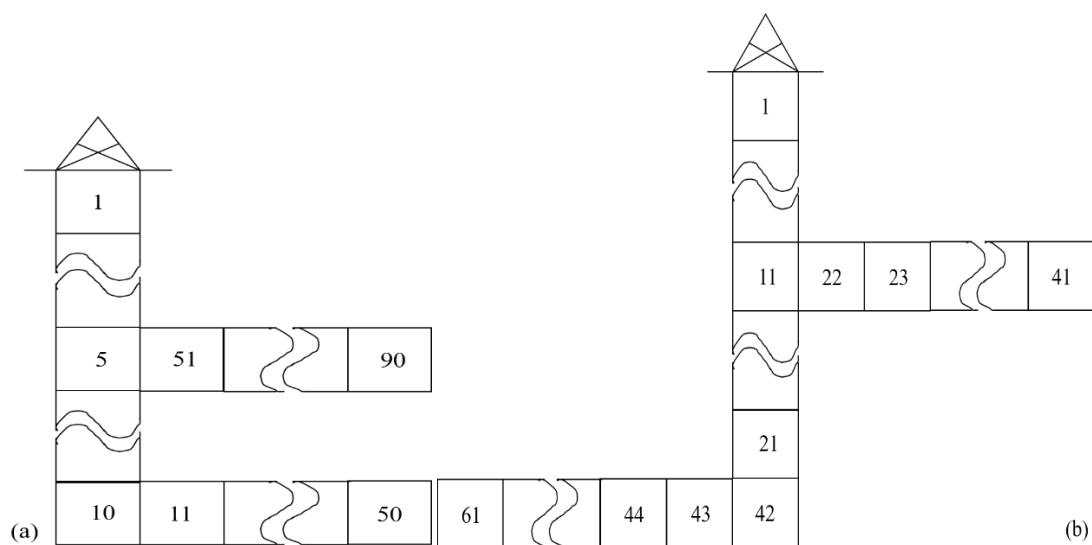


Figure 10. Schematic representation of multilateral wells: (a) dual-lateral well; (b) multilateral well with three branches. This figure was modified from Figure 2 in ref (Livescu et al., 2010a) and Figure 6 in ref (Livescu et al., 2008).

5. Summary

In this study, we conduct a literature assessment of the dynamic interactions between wellbores and reservoirs. We explain the wide application of the DF model in the wellbore model for coupling the reservoir models. We honor the fully implicit method for coupling the reservoir system and wellbore system due to its high stability and accuracy. It should be emphasized that dynamic coupled reservoir/wellbore simulators are necessary for resolving transient flow problems between reservoir and wellbore.

Livescu et al., 2010a; Livescu et al., 2008; Pourafshary, 2007; Cao et al., 2015; Shirdel, 2013

developed their own coupled reservoir/reservoir models for general scenarios of different wellbore types with advanced completions. The findings of the numerical analysis of pressure, temperature, mole fractions, and in-situ phase fractions matched well with experimental data or analytical solutions.

In horizontal or deviated wells, gas coning phenomenon reflected typical dynamic interactions between reservoir and wellbore. Belfroid et al., 2005; Leemhuis et al., 2008; Nennie et al., 2008 noticed a lower bottom hole pressure and an earlier simulated gas breakthrough time in their dynamically coupled simulator, which performed slightly different simulation results from a standalone reservoir simulator.

During start-up and shut-down, the well exhibited robust dynamics. Alberts et al., 2007; Nennie et al., 2008 showed that the bottom hole pressure reached the reservoir pressure more slowly and is smaller than the initial reservoir pressure in the coupled model during the shut-in period. Forouzanfar et al., 2015 captured the clear wellbore storage effect by demonstrating an unstable constant pressure derivative value during the radial flow in the draw-down test. Bahonar et al., 2011 reported that the coupled model performed better than the commercial well testing simulator in temperature impact and wellbore storage effect.

Tang et al., 2019a; Tang et al., 2019b; Tang et al., 2017 accurately anticipated the gas rate production and pressure of the perforated segment during the liquid loading period. Their coupled model observed the concise flow-pattern transition and forecasted the onset of the liquid loading one day earlier than E300.

We note, more coupled simulators will be utilized to address other transient flow phenomena to further define the necessity of coupling reservoir/wellbore modeling.

Authorship contribution statement

Long Peng: Investigation, Writing-original draft. **Guoqing Han:** Supervision, Funding acquisition, Writing- review & editing. **Zhangxin Chen:** Supervision, Writing- review & editing. **Arnold Landjobo Pagou:** Investigation, editing. **Liyang Zhu:** Investigation, review.

501 **Akhayie Mamat Abdoulaye:** Review.

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