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Research on Seepage Modeling of Volume Fracturing Horizontal Wells in Fractured Stress-Sensitive Tight Oil Reservoirs

Zhong Li¹, Xinjiang Yan¹, Min Wen¹, Gang Bi^{2,*}, Nan Ma¹ and Zongxiao Ren¹



¹ CNOOC Research Institute Co. Ltd Beijing Chaoyang, China; lizhong@cnooc.com.cn

² College of Petroleum Engineering, Xi'an Shiyou University, Xi'an, China; big@xsyu.edu.cn

* Correspondence: big@xsyu.edu.cn; Tel.: (+86-15091759039)

Abstract: Tight oil reservoirs tend to contain more natural fractures, and the presence of natural fractures leads to a greater stress sensitivity in tight oil reservoirs. It is a very challenging task to model the seepage in the volume fracturing horizontal wells considering the stress-sensitive effects. Based on the Laplace transform, Perturbation transform and Stefest numerical inversion, this paper establishes a horizontal well seepage model for volume fracturing in fractured stress-sensitive tight oil reservoirs. This model allows to analyze and study the effect of stress sensitivity, fracture interference, dual media and complex fracture network on seepage flow in tight oil reservoirs. We apply the model to delineate the seepage stages of volume fracturing horizontal wells in tight oil reservoirs and analyze the sensitivity of the main parameters such as stress sensitivity, storage capacity ratio, skin factor and wellbore storage, which can provide theoretical guidance for the efficient development of tight oil reservoirs.

Keywords: Tight oil reservoir; seepage model; stress sensitive; volume fracturing horizontal well; interfracture interference

1. Introduction

With the gradual decrease of conventional energy sources, the development of tight oil reservoirs, as one of the important alternative energy sources, has attracted the attention of oil companies worldwide. Compared with the numerical simulation of conventional reservoirs, the simulation of seepage in tight oil reservoirs is more complicated. The reasons are as follows: dense reservoir rocks are highly brittle and contain a large number of natural fractures, leading to the formation of complex fracture networks easily during large-scale hydraulic fracturing [1]. As early as the 1950s, foreign scholars studied the stress-sensitive phenomenon in the formation [2–4]. In 1971, Vairogs [5] conducted stress-sensitive experiments on cores with different initial permeability, and the results showed that the lower the initial permeability the greater the stress sensitivity of the rock, and the appearance of natural fractures and shale streaks exacerbated the stress-sensitive phenomenon in the cores. Tight oil reservoirs have very low initial permeability and contain a large number of natural fractures, and are therefore highly stress sensitive. A large number of seepage models for volume fracturing horizontal wells in tight oil reservoirs have been established by domestic and foreign scholars, and these models can be classified into three categories: analytical models, semi-analytical models and numerical models. This paper focuses on the semi-analytical model, and the semi-analytical seepage model for volume fracturing horizontal wells is described in detail below. Semi-analytic models are established based on source functions, and there are three source functions currently used in petroleum engineering to describe oil reservoir seepage: Gringarten source function [6], Ozkan dual

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medium source function, and Valko volume source function [7]. Gringarten source function and Valko volume source function can only simulate single heavy media oil reservoir seepage problems, and Ozkan source function is more suitable for solving seepage problems in tight oil reservoirs containing a large number of natural fractures. Scholars have used the Ozkan source function as a theoretical basis to model seepage in a large number of single volume fracturing horizontal wells in tight oil reservoirs [8–15]. In order to study the seepage pattern of multiple volume fracturing horizontal wells, domestic and foreign scholars [16–18] have developed mathematical models of seepage when multiple volume fracturing horizontal wells are produced simultaneously.

However, none of the above models can consider the effect of stress sensitivity of tight oil reservoirs on the seepage of the volume fracturing horizontal wells, and the subsurface fluid continuity equation will become a strongly nonlinear partial differential equation after considering stress sensitivity, which is much more difficult to solve. In this paper, we transform the nonlinear partial differential equation into a linear partial differential equation by using Perturbation transform, and then apply Laplace transform, superposition principle and the “star-triangle” transform to establish a mathematical model of the seepage in volume fracturing horizontal well. Finally, we apply Gauss-Jordan elimination method to obtain the solution of the mathematical model of seepage in Laplace space, and transform the Laplace space solution into a time-space solution by Stehfest (1970) numerical inversion. Using this model, we have systematically studied parameters such as stress sensitivity, capacity storage ratio, skin factor, wellbore storage effect and channeling flow coefficient, etc. The above research results are of great significance to the efficient exploitation of tight oil reservoirs.

2. Mathematical modeling

The Materials and Methods should be described with sufficient details to allow others to replicate and build on the published results. Please note that the publication of your manuscript implicates that you must make all materials, data, computer code, and protocols associated with the publication available to readers. Please disclose at the submission stage any restrictions on the availability of materials or information. New methods and protocols should be described in detail while well-established methods can be briefly described and appropriately cited.

Assuming that the volume fracturing horizontal wells in tight reservoirs contain 24 fractures (as shown in Figure 1), the fracturing horizontal well seepage flow can be divided into three parts: reservoir seepage flow, flow within the complex fracture network, and horizontal wellbore variable mass tubular flow. Due to the small pressure drop of the variable mass tubular flow in the horizontal wellbore, it is ignored here.

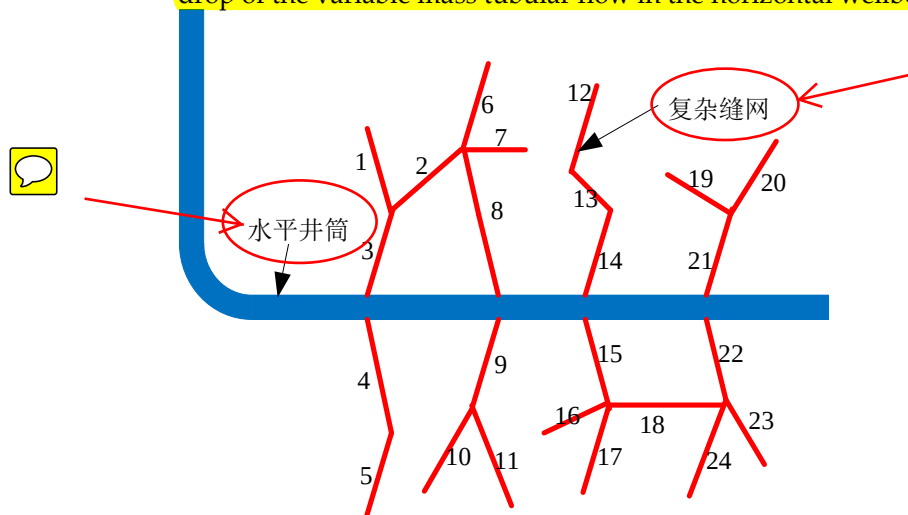


Figure 1. Schematic diagram of volume fracturing horizontal well.

2.1. Mathematical model of oil reservoir seepage

Tight oil reservoirs are dual media reservoirs, consisting of a natural fracture system as well as a matrix system, and the equation of fluid continuity in the natural fracture system is :

$$\frac{k_f}{\mu} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Delta p_f}{\partial r} \right) - (V\phi c)_m \frac{\partial \Delta p_m}{\partial t} = (V\phi c)_f \frac{\partial \Delta p_f}{\partial t}, \quad (1)$$

Assuming a proposed steady-state channeling flow between the matrix system and the natural fracture system, the matrix system seepage control equation is:

$$(V\phi c)_m \frac{\partial \Delta p_m}{\partial t} = \sigma \frac{k_m}{\mu} (\Delta p_f - \Delta p_m), \quad (2)$$

Considering the stress sensitivity of the natural fracture system, the natural fracture permeability can be expressed as [19]:

$$k_f = k_{if} e^{-\alpha(p_i - p_f)}, \quad (3)$$

The single fracture source function is obtained from Perturbation transform, Laplace transform, and the mirror principle as [20]:

$$\bar{\eta}_{D0} = \bar{q}_{fD} \int_{x_{wD}-l_{fD}}^{x_{wD}+l_{fD}} K_0 \left(\sqrt{(x_{wD}-\xi)^2 + (y_D - y_{wD})^2} \sqrt{sf(s)} \right) d\xi, \quad (4)$$

Equation (4) can only solve the fracture vertical horizontal wellbore seepage problem. From the principle of coordinate translation and coordinate transformation, we can obtain the source function for the existence of any angle between the fracture and the horizontal wellbore.

$$\bar{\eta}_{D0} = \bar{q}_{fD} \int_{-l_{fD}}^{l_{fD}} K_0 \left[\sqrt{sf(s)} \sqrt{[(x_D - x_{fD}) \cos \theta + (y_D - y_{fD}) \sin \theta - u]^2 + [(y_D - y_{fD}) \cos \theta - (x_D - x_{fD}) \sin \theta]^2} \right] du, \quad (5)$$

Considering that fractures interfere with each other within the volumetric fracture network, according to the superposition principle [21], the expression for the dimensionless pressure drop for any one fracture in the volume fracturing horizontal well schematic (Figure 1) is of the following form:

$$\bar{\eta}_{D0,i} = \sum_{j=1}^{24} \bar{q}_{fDj} \bar{\eta}_{D0i,j} \quad i = 1, 2, \dots, 24, \quad (6)$$

The resistance to seepage without considering fractures is:

$$\bar{\eta}_{D0,i} = \bar{\eta}_{wD} \quad i = 1, 2, \dots, 24, \quad (7)$$

Limitation requirement for the sum of the dimensionless yields of 1 for each fracture:

$$\sum_{j=1}^{24} \bar{q}_{fDj} = 1/s, \quad (8)$$

2.2. Mathematical model of seepage within complex fracture networks

Assuming that the fluid in the fracture satisfies the one-dimensional Darcy seepage, the expression for the seepage in the fracture is given by the “star-triangle” transformation method [22]:

$$\bar{q}_{Df,i} = K_{i,j}(\bar{p}_{Dj} - \bar{p}_{Di}), \quad (9)$$

$$K_{i,j} = \frac{T_{ij}\mu}{2\pi kh} \quad T_{i,j} = \frac{\gamma_i\gamma_j}{\sum_{k=1}^n \gamma_k} \quad \gamma_i = \frac{Ak_i}{\mu D_i}, \quad (10)$$

Then the fracture seepage matrix within the complex fracture network in Figure 1 is shown below:

$$K_p = q, \quad (11)$$

$$\mathbf{p}^T = [\bar{\eta}_{D0,1} \quad \bar{\eta}_{D0,2} \quad \bar{\eta}_{D0,3} \quad \cdots \quad \bar{\eta}_{D0,24}] \quad \mathbf{q}^T = [\bar{q}_{D1} \quad \bar{q}_{D2} \quad \bar{q}_{D3} \quad \cdots \quad \bar{q}_{D24}] \quad (12)$$

2.3. Mathematical model for coupled seepage in volume fracturing horizontal wells of tight oil reservoirs

Combining Eq. (6), Eq. (7), Eq. (8), and Eq. (11), the continuity condition of equal pressure and flow at the boundary yields:

$$\begin{bmatrix} \mathbf{A} & -\mathbf{I} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ -\mathbf{I} & \mathbf{K} & \mathbf{O}_3 & \mathbf{O}_4 & \mathbf{O}_8 & \mathbf{O}_9 & \mathbf{O}_{14} & \mathbf{O}_{15} & \mathbf{O}_{21} & \mathbf{O}_{22} & \mathbf{O} \\ \Theta & \Theta & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ \Theta & \Theta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ \Theta & \Theta_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \mathbf{p} \\ \bar{Q}_{D3} \\ \bar{Q}_{D4} \\ \bar{Q}_{D8} \\ \bar{Q}_{D9} \\ \bar{Q}_{D14} \\ \bar{Q}_{D15} \\ \bar{Q}_{D21} \\ \bar{Q}_{D22} \\ \bar{\eta}_{wD} \end{bmatrix} = \begin{bmatrix} \mathbf{O} \\ \mathbf{O} \\ 1/s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (13)$$

$$\mathbf{A} = \begin{bmatrix} \bar{\eta}_{D01,1} & \bar{\eta}_{D01,2} & \cdots & \bar{\eta}_{D01,24} \\ \bar{\eta}_{D02,1} & \bar{\eta}_{D02,2} & \cdots & \bar{\eta}_{D02,24} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\eta}_{D024,1} & \bar{\eta}_{D024,2} & \cdots & \bar{\eta}_{D024,24} \end{bmatrix} \quad \Theta = \mathbf{O}^T = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{24 \times 1} \quad \mathbf{I} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{24 \times 1}, \quad (14)$$

Eq. (18) is the volume fracturing horizontal well capacity model in Fig. 1. There are 24 fractures in the fracture network, and the unknowns include $\bar{q}_{Di}$ ($i=1,2,\dots,24$), $\bar{\eta}_{D0,i}$ ($i=1,2,\dots,24$), $\bar{Q}_{D3}$, $\bar{Q}_{D4}$, $\bar{Q}_{D8}$, $\bar{Q}_{D9}$, $\bar{Q}_{D14}$, $\bar{Q}_{D15}$, $\bar{Q}_{D21}$, $\bar{Q}_{D22}$ and $\bar{\eta}_{wD}$, a total of 57. The number of equations in the matrix is also 57, so the system of equations is solvable, and the solution of the system of equations can be obtained by the Gauss-Jordan elimination method.

The expression for the pressure drop at the bottom of the well considering stress sensitivity is obtained from the Perturbed inverse transformation:

$$\bar{p}_{wD} = -\frac{\ln(1 - \alpha \bar{\eta}_{wD})}{\alpha}, \quad (15)$$

Eq. (15) is the formula for the dimensionless bottomhole pressure drop without considering the wellbore storage and skin coefficient, and the formula for the bottomhole pressure drop considering the wellbore storage and skin coefficient is :

$$\bar{p}_{wD} = \frac{s \bar{p}_{wD} + S}{s + C_{wD} s^2 (s \bar{p}_{wD} + S)}, \quad (16)$$

The solution of Eq. (16) in time space can be obtained by using the numerical inversion of Stehfest [23].

3. Seepage staging of volume fracturing horizontal well in tight oil reservoir

Taking the volume fracturing horizontal well in Figure 1 as an example, other data for tight oil reservoirs are summarized as shown in Table 1.

Table 1. Basic data of tight oil reservoir.

Parameters	Values	Unit
Thickness of oil reservoir	10	m
Permeability in x-direction	0.01	mD
Permeability in y-direction	0.01	mD
Permeability of fracture	1700	mD
Channeling flow coefficient	10 ⁻⁶	Real number
Initial pressure of oil reservoir	12	MPa
Well production	15.9	m ³ /d
Porosity of oil reservoir	0.1	Real number
Volume coefficient	1	m ³ /m ³
Viscosity of crude oil	1	mPa.s
Compression coefficient	4.35×10 ⁻⁴	MPa ⁻¹
Storage coefficient	0.03	Real number
Stress sensitivity coefficient	0.1	Dimensionless
Wellbore storage coefficient	0.8	Dimensionless
Skin coefficient	0.5	Dimensionless

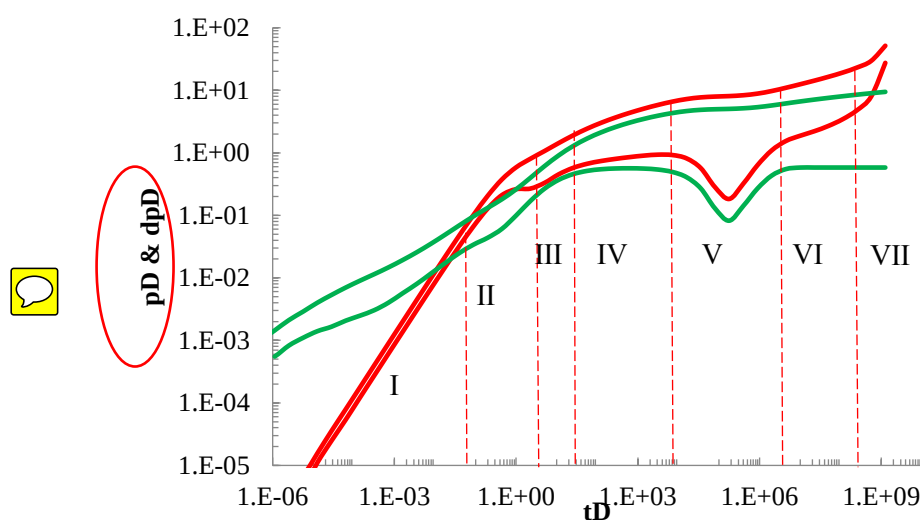


Figure 2. Well test curve of volume fracturing horizontal well in tight oil reservoir.

The test curves for fracturing horizontal wells in both cases are given in Figure 2. The green line is the test curve when stress sensitivity, wellbore storage, and well skin are not considered, and the red line is the test curve when these factors are considered. The well test curve of the first case can be divided into seven seepage stages, which are: linear flow within the fracture, fracture-formation bilinear flow, formation linear flow, transition flow, natural fracture system proposed radial flow, scramble flow, and system proposed radial flow. The well test curve for the second case can be similarly divided into seven seepage stages (as shown in Figure 2): I wellbore storage flow, II surface flow stage, III transition flow, IV natural fracture system proposed radial flow, V scramble flow, VI system proposed radial flow, and VII stress-sensitive flow stage, respectively. Comparing the well test curves for the two cases in Figure 2, when considering wellbore storage, well skin, and stress sensitivity, the wellbore storage flow and skin flow phases mask the in-fracture linear flow, fracture-formation bilinear flow, and formation linear flow; the slope of the dimensionless pressure drop derivative in the well test curve is 1. The values of pressure drop and pressure drop derivative of IV natural fracture system proposed radial flow, V channeling flow, VI system proposed radial flow and VII stress-sensitive flow stage gradually increase due to the reservoir stress sensitivity. It should be noted that the well pressure drop when considering wellbore storage is lower than when not considering wellbore storage, indicating that wellbore storage is a beneficial factor for well production.

4. Sensitivity analysis

To further clarify the influence of each factor on the seepage of volume fracturing horizontal wells, we performed sensitivity analysis on key parameters such as stress sensitivity factor, wellbore storage factor, well skin factor, and storage capacity ratio.

4.1. Stress sensitivity factor analysis

The oil reservoir base parameters are shown in Table 1, and when the stress sensitivity coefficients are 0.05, 0.08, and 0.10, respectively, the well test curves change as shown in Figure 3. As can be seen from Figure 3, the stress sensitivity factor has no effect on the early seepage, and mainly affects the late stage of well exploitation. The larger the stress sensitivity factor is, the larger the upwarping amplitude of the late well test curve is.

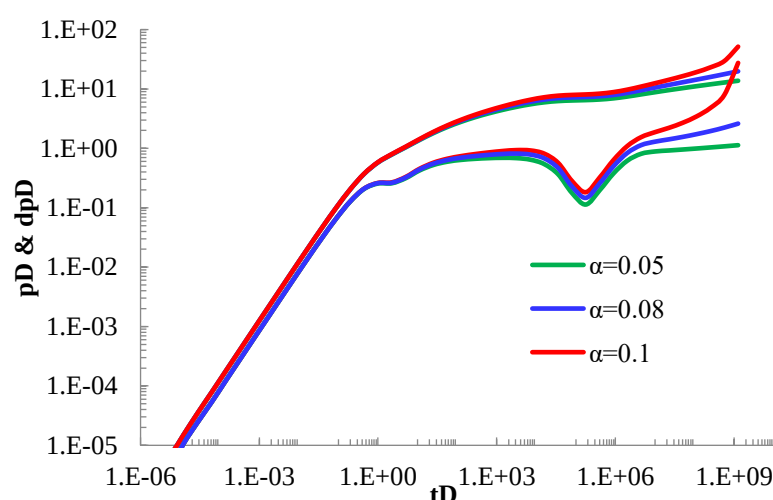


Figure 3. Well test curve of formation stress sensitivity analysis.

4.2. Wellbore storage factor sensitivity analysis

The oil reservoir base parameters are shown in Table 1, and when the wellbore storage coefficients are 0.8, 2, and 4, respectively, the well test curves change as shown in Figure 4. From Figure 4, it can be seen that the wellbore storage coefficient mainly affects the pre-well seepage flow and has no effect on the post-seepage flow. The larger the wellbore storage coefficient, the longer the duration of the wellbore storage flow phase, even masking the well skin flow phase.

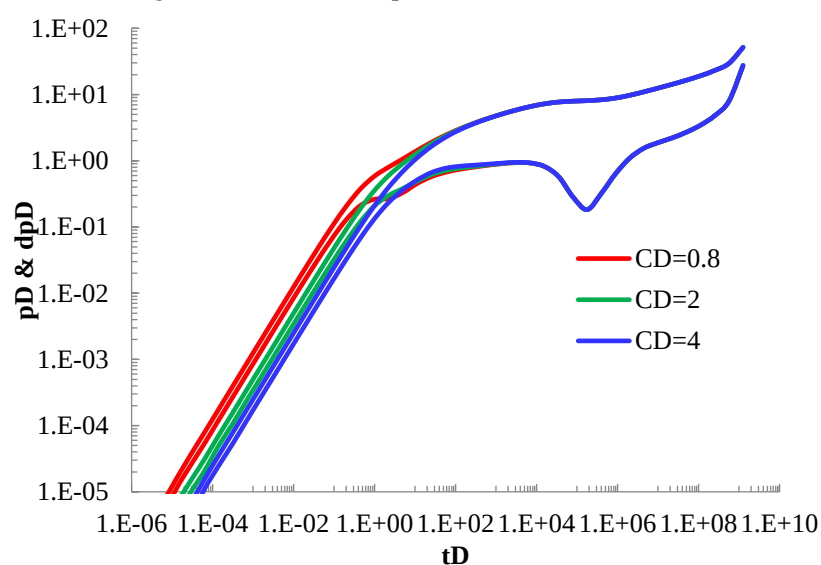


Figure 4. Well test curve of wellbore storage coefficient sensitivity analysis.

4.3. Sensitivity analysis of skin coefficients

The oil reservoir base parameters are shown in Table 1, and when the skin coefficient is 0.5, 1, and 2, respectively, the well test curve changes as shown in Figure 5. As can be seen from Figure 5, the skin coefficient has a greater effect on the later seepage, and the larger the skin coefficient the greater the well pressure drop and the larger the upwarping amplitude of the dimensionless pressure drop derivative curve.

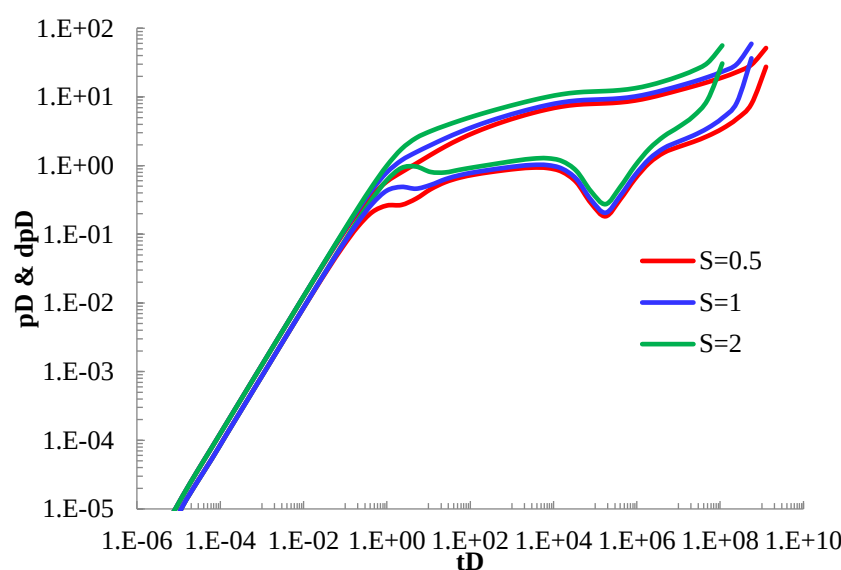


Figure 5. Well test curve of skin coefficient sensitivity analysis.

4.4. Storage capacity ratio sensitivity analysis

The oil reservoir base parameters are shown in Table 1, and when the reservoir storage capacity ratio is 0.03, 0.1, and 0.3, respectively, the well test curves change as shown in Figure 6. The storage capacity ratio mainly affects the seepage before the occurrence of channeling flow, but has no effect on the seepage after the occurrence of channeling flow. The larger the storage capacity ratio, the shallower the “notch” in the channeling stage.

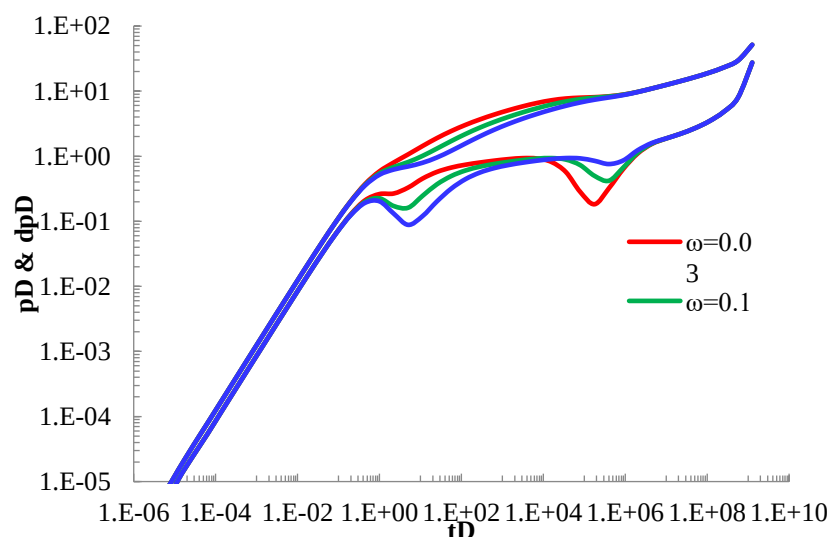


Figure 6. Well test curve of storage capacity ratio sensitivity analysis.

4.5. Sensitivity analysis of channeling flow coefficient

The oil reservoir base parameters are shown in Table 1, and when the channeling flow coefficient is 10^{-6} , 10^{-4} , and 10^{-2} , respectively, the well test curves change as shown in Figure 7. As can be seen from Figure 7, the channeling flow coefficient mainly affects the timing of fugacity between the matrix system and the natural fracture system. The larger the channeling flow coefficient is, the earlier the channeling flow occurs between the two systems and the more the “notch” position is to the left.

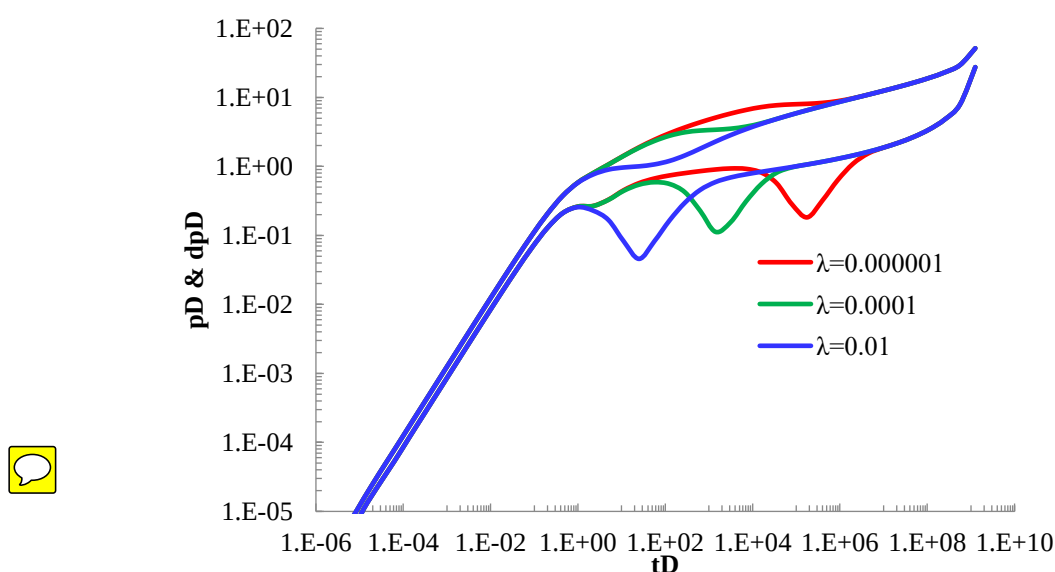


Figure 7. Well test curve of channeling flow coefficient sensitivity analysis.

5. Conclusions

In this paper, we developed and solved a seepage model for volume fracturing of horizontal wells in tight oil reservoirs using Perturbation transform, Laplace transform, Gauss-Jordan elimination method, and Stehfest numerical transform. The model considers the effects of formation stress sensitivity, interfracture interference, wellbore storage coefficient, and well skin coefficient on horizontal well seepage. The following insights were gained in the course of the study.

(1) Stress-sensitive tight oil reservoir volume fracturing horizontal wells can be divided into seven seepage stages: I wellbore storage flow, II skin flow stage, III transition flow, IV natural fracture system proposed radial flow, V channeling flow, VI system proposed radial flow, and VII stress-sensitive flow stage. Wellbore storage flow stages conceal fracture linear flow, fracture-formation bilinear flow, and formation linear flow seepage characteristics.

(2) The wellbore storage factor mainly affects the wellbore storage phase flow, the larger the wellbore storage factor, the longer the duration of the wellbore storage flow. The well skin has a greater impact on later development; the larger the skin coefficient, the greater the well pressure drop.

(3) Formation stress sensitivity mainly affects the seepage characteristics of wells in the late development period. The larger the stress sensitivity coefficient is, the larger the upwarping amplitude of the dimensionless pressure drop derivative curve is.

(4) The storage capacity ratio has a relatively large impact on the flow before the occurrence of channeling flow, and the depth of the “notch” on the pressure drop derivative curve becomes shallower as the storage capacity ratio increases. The channeling flow coefficient mainly affects the timing of the channeling flow phase. The larger the channeling flow coefficient, the earlier the channeling flow of the matrix system to the natural fracture system, and the more to the left the “notch” position.

Parameter definition

A fracture intersection area, m^2 ; C wellbore storage coefficient; D distance between two fracture centers, m; h oil reservoir thickness, m; k permeability, μm^2 ; k_{if} initial fracture permeability, μm^2 ; l fracture length, m; p pressure, Pa; p_i initial formation pressure, Pa; Q horizontal well flow rate, m^3/s ; r radius, m; q flow rate, m^3/s ; s Laplace variables; S well skin; T interfracture conductivity; α stress sensitivity coefficient, fractional; c rock compression coefficient, pa^{-1} ; ϕ porosity, fractional; μ crude oil

viscosity, mpa s; θ fracture dip angle, °; ω storage capacity ratio, fractional; λ channeling flow coefficient, fractional.

Subscript

w represents horizontal wellbore; D dimensionless variable; m matrix system; f natural fracture system.

Superscript

" " Laplace space variable.

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