Dear Editor,

thank you very much for your email and for the opportunity to resubmit the paper in a revised version that incorporates/receives the Reviewer(s) suggestions. We examined in detail the Reviewers' comments and we feel that all the points raised by them could be addressed to both better explain some aspects of the work and clarify all the issues at hand.

Below, all the replies to the Reviewers' comments are point-by-point reported, by highlighting – as requested – all the changes/amendments considered in the revised version of the manuscript (in black the original Reviewers comments/questions, in red the Authors responses).

REVIEWER 1

In this paper, the authors conducted a comprehensive literature survey related to the classical and the first- and higher-order shear deformation theories of functionally graded (FG) beams, for which the governing equations and analytical solutions were presented. The modified couple stress effects and the von Karman nonlinearity effects were also considered in the formulation of this paper.

Some suggestions are given as follows:

- The authors may explain the difference between the boundary conditions of the FG beams in the modified couple stress beam theories and those in the classical continuum mechanics-based beam theories.

In the revised version of the paper has been added the comment:

$$\frac{dN_{xx}}{dx} + f = 0, \tag{13}$$

$$\frac{d^2\bar{M}_{xx}}{dx^2} + \frac{d}{dx}\left(\frac{dw}{dx}N_{xx}\right) + q = 0, \tag{14}$$

where $\bar{M}_{xx} = M_{xx} + P_{xy}$ takes into account the modified couple stress effects in both the governing equations and the boundary conditions.

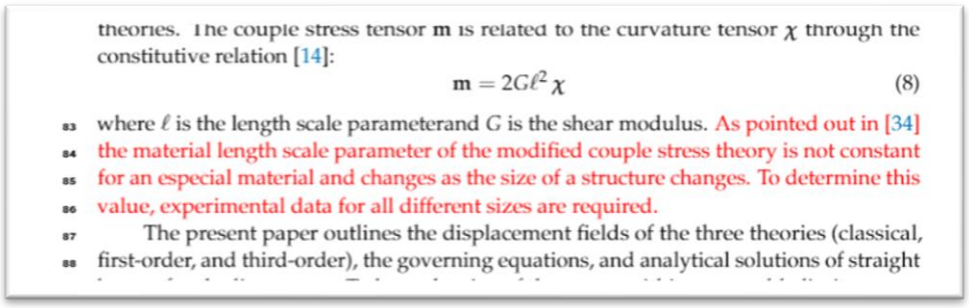
The duality pairs of the CBT are (the first element of each pair is the primary variable and the second element is the secondary variable)

$$(u, N_{xx}), \quad (w, V_{\text{eff}}), \quad \left(-\frac{dw}{dx}, \bar{M}_{xx}\right). \tag{15}$$

where V_{eff} is the effective shear force

- The authors may explain how to obtain the material length scale parameter (*l*) when the couple stress effects were considered.

In agreement with the reviewer's comment, in the revised version of the paper has been added the new remark:



theories. The couple stress tensor $\mathbf{m}$ is related to the curvature tensor χ through the constitutive relation [14]:

$$\mathbf{m} = 2G\ell^2\chi \tag{8}$$

where ℓ is the length scale parameterand G is the shear modulus. As pointed out in [34] the material length scale parameter of the modified couple stress theory is not constant for an especial material and changes as the size of a structure changes. To determine this value, experimental data for all different sizes are required.

The present paper outlines the displacement fields of the three theories (classical, first-order, and third-order), the governing equations, and analytical solutions of straight

- The paper is well organized and well written. I strongly recommend that it should be accepted by *Applied Sciences*. The above suggestions are given for the authors' reference, and the second run of review is no need.

The authors thank the reviewer for his/her positive comment on our manuscript.