Response to the reviewers

Reviewer 3

The article deals with the conversion of existing Euler-Bernoulli analytical relationships to equivalent Timoshenko ones to improve the accuracy of analytical equations for the mechanical properties of repeatable metamaterials, while the technique has been validated at several cell types, so-called lattice structures.

I rate the manuscript positively. It provides a comprehensive view of the issue, and it can be said that the team of authors has an overview of the theories and results achieved by other scientists and experts in this field.

Although I did not find any significant shortcomings, I have a few comments on how to improve it:

1. From a methodological point of view, it would be more appropriate for the authors to first present the types of individual cells (as the "Material" in the section "2. Materials and Methods"), and so provide to readers with their characteristics (cell size, strut diameter, ...), ... and only then focus on methods, as the methods refer to the structures.

Answer: Since the main goal of this study is to provide an analytical methodology to improve the accuracy of (already-existing) analytical solutions for the strut-based lattice structure, the authors decided to analyze six well known lattice structures as case studies for validation of the analytical methodology which has been presented in the “Materials and Methods” section. More detailed information on the dimensions and size of the unit cells is now added to the Section 2.3 as follow:

> “It must be noted that the commonly known geometries for the six noted unit cell types are presented in Figure 4 and the actual geometries used for both the analytical and numerical analyses in this study are given in Figure 5. The reason for such conversion of unit cell shape for BCC is obvious as the molecular structure of BCC (Figure 4a) is composed of spheres rather than struts (Figure 5a). For the case of diamond, the unit cell shown in Figure 4b is rotated, and the unit cell in a specific direction (for which all the struts have similar angles with respect to horizontal plane) has been considered for analysis (Figure 5b). For the other cases, the unit cell conversion has been made in order to avoid neighboring cells having adjacent struts. If the neighbor cells have adjacent side-by-side edges, the analytical relationships obtained for the unit cell does not represent that of a lattice structure. More explanation regarding this can be found in Section 2.1.2 of [32]. Although, in this paper, all analytical solutions for the lattice structures and unit cells are presented in a normalized manner, and hence the dimensions of the unit cells do not affect the results, an equal volume of $5 \times 5 \times 5$ mm3 was considered for all the unit cell types. Moreover, in each unit cell, the struts radius was increased from very small values ($r \cong 0$) to high values up to the point where relative density of the unit cell reached $\mu = 0.5$.”

Extra detailed information on the topology of the structures could be found in the cited references which have been mentioned in the manuscript.

2. From the point of view of better orientation in the text, it would be appropriate for the authors to arrange the pictures so that their order/numbering is in the order mentioned in the text (for example, a reference to Figure 1 is given in the text only after references to Figures 2 and 3, ... it is also similar with references to Figures 2b and 2c).

Answer: The following paragraph was added to the beginning of the section "2. Materials and Methods" and the order of Figure 1 has been modified.

"The general deformation of any arbitrary strut of an open-cell lattice structure can be decomposed into four basic deformations (Figure 1). All of the struts of a unit cell could be considered as a clamped beam with four main deformations namely lateral displacement (v), flexural rotation (θ), twist (φ), and elongation/contraction (u). By considering these deformations as basic deformations of the struts of a lattice structure, the total deformation and behaviour of a unit cell in lattice structure could be calculated and obtained. In this section, the basic formulation of deformation for a single strut of unit cell under various conditions based on two well-known beam theories (Euler-Bernoulli and Timoshenko) are obtained."

3. The authors should provide a broader discussion regarding the critical points for each unit cell type in Figure 9.

Answer: A discussion subsection (Section 4.1) related to the behaviors of the unit cells and the critical points is now added to the manuscript:

"4.1. Unit cells behaviour

The main mechanism of deformation in the struts of lattice structures and porous materials is flexure, stretching/contraction, or a combination of them. In the unit cells with a high fraction of vertical struts (struts aligned with the loading condition), the main reason for collapse is the axial normal stresses generated in their struts. Therefore, the deformation in these structures is stretching-dominated and the lattice structure collapses due to generation of unbearable axial stress in the struts. As it can be seen in Figure 9, the hexagonal packing and truncated cube unit cells could be considered as stretching-dominated structures due to the high presence of vertical struts in their architecture, while the other structures namely BCC, diamond, truncated octahedron as well as rhombicuboctahedron could be considered as bending-dominated structure, as they are mostly made up of oblique struts. The main mode of deformation in a unit cell (stretching-dominated or bending-dominated) determines the general deformation of the unit cell and, therefore, its stiffness and yield strength. According to Figure 9, the critical points of the hexagonal packing and truncated cube unit cells are located in the vertical struts due to their stretching-dominated behaviour. On the other hand, the critical points of BCC, diamond, truncated octahedron, and rhombicuboctahedron unit cells are located at the end of oblique struts due to their bending-dominated behaviour. In addition, it can be seen in Figure 6 and Figure 8 that among all the unit cell types, the hexagonal packing and truncated cube structures have the highest stiffness and yield strength, especially in lower values of relative density due to their stretching-dominated behaviour. It is worth noting that since the rhombicuboctahedron has vertical struts, it has an in-between behaviour and gives higher stiffness and strength in comparison with other bending-dominated unit cells. More figures which illustrate the axial and bending stresses in the struts of the unit cells could be found in Appendix B."

Moreover, the following sentences are now added to Section 4.2:

"It is worth mentioning that in the bending-dominated unit cells, the discrepancy between the Euler-Bernoulli and Timoshenko results is much higher as compared to stretch-dominated unit cells (Figure 6). In other words, the inability of Euler-Bernoulli beam theory to predict the effective

elastic moduli of open cell structures based on BCC, diamond, and truncated octahedron unit cells is attributed to the their main mode of deformation (bending-domination) and the importance of considering the shear deformation effect."

4. It would be more appropriate to divide the individual relations given as equation (10) into separate lines.

Answer: The equation (10) shows the integration sequence and does not provide any separate and individual relation. Hence, we keep it as a single equation and just summarize the integration details. Equation (10) is now summarized into one line.

5. References to equations should be enclosed in parentheses in the text, not in quotation marks.

Answer: The equations mentioned in quotation marks were actually references to equations in other articles, and they were not the achievements of this paper. The authors had decided to refer to such already-existing equations to make the procedure of deriving new equations more understandable for the reader. Appendix D which lists equations extracted from the literature and used in this study is now added to the manuscript. The equations which were in quotation marks are now referred to equations in Appendix D.

6. The conclusions should be formulated more specifically in light of the results achieved.

Answer: The following sentences are now added to Conclusions section to make the results of the study more visible in Conclusion section:

"The highest improvement in the analytical relationships was observed in bending-dominated structures such as BCC, diamond, and truncated octahedron topologies and especially in higher relative densities, where shear deformation effect becomes significant. While the conversion methodology introduced in this study had a significant effect on improvement of elastic modulus and Poisson's ratio relationships, its effect on yield stress relationships was much lower, and in some cases (such as BCC, diamond, and truncated cube) negligible."

7. On page 7, line 237, it would be appropriate to highlight that these are the Figures a, b, e, f in Annex B, similarly as it is in line 246 on the same page.

Answer: The term is now changed to the following:

"(Figure Ba,b,e,f in Appendix B)"

8. As the main text of the article refers to Annexes A to C, this should be a publishable part of it accessible to all readers (it should not be just additional documents provided by reviewers).

Answer: Appendices A-E are called "Appendix" and they are not called Supplementary materials. Therefore, they will be part of the final section of the main paper.