

## Reply to Reviewer 2 Comments

The authors want to take this opportunity to express their sincere gratitude to the editor and the reviewers for their valuable instructions on revising the paper. A point-by-point response to the editorial and reviewer comments is listed below.

### Reviewer #2 comments:

- (1a) Despite its crucial importance and extensive use throughout the paper, the probability distribution  $\rho_c(t, x, y)$  does not seem to be defined explicitly anywhere in the text. The authors must provide the definition by providing an equation to clarify what it is.

Reply: Thank you for your reminding. We add the definition of the join probability  $\rho_c(t, x, y)$  in Equation (53), which is the solution of the Fokker-Planck equation in the  $x - y$  plane:

$$\frac{\partial \rho_c}{\partial t} = -\frac{\partial(v_R \rho_c)}{\partial x} - \frac{\partial(v_I \rho_c)}{\partial y} + \frac{D^2}{2} \left( \frac{\partial^2 \rho_c}{\partial x^2} - 2 \frac{\partial^2 \rho_c}{\partial x \partial y} + \frac{\partial^2 \rho_c}{\partial y^2} \right)$$

The related description in the paper please read from line 378 to line 382 at page 12.

- (1b) Related to the above, the authors should give the reason why the quantity  $\rho_c(t, x, y)$  qualifies as probability distribution. Is it simply because the marginal distribution defined in (59) becomes a probability distribution over the space  $x$ ?

Reply: The joint probability  $\rho_c(t, x, y)$  presents the probability of finding the particle in the complex plane at position  $z = x + iy$  at time  $t$ . The qualification of  $\rho_c(t, x, y)$  as a probability distribution is guaranteed by the fact that it is the solution to the Fokker-Planck equation. The related description in the paper please read from line 378 to line 382 at page 12.

- (1c) There are portions which look somewhat sloppy in the text. For instance, the same notation for the velocity  $v_B$  is used both in (5) and (7) but obviously they are contradictory to each other. Also, in (41) a derivative (in the difference form) appears to be missing in the second term of the r.h.s. containing  $\psi_n(t, z)$ . These should be corrected appropriately.

Reply: Thank you for reminding this mismatch notation of the velocity  $v_B$ . We have modified it. We use  $v_{BM}$  in Equation (5) instead of  $v_B$ , also in line 198.

- (1d) How can this proposed stochastic model defined by making the one dimensional space complex be generalized to the three spatial dimensions where we live? The authors are advised to comment on this in the last concluding section.

Reply: This is a good question. We can have the stochastic model for three dimensional complex space by considering the quantum particle with random motion under the optimal guidance

control law. The stochastic differential equations for a quantum particle in the complex spherical coordinate system  $r = r_R + ir_I$ ,  $\theta = \theta_R + i\theta_I$ ,  $\phi = \phi_R + i\phi_I$  can be expressed as

$$dr = -\frac{i\hbar}{m}\left(\frac{\partial \ln \Psi}{\partial r} + \frac{1}{r}\right)dt + \sqrt{\frac{\hbar}{2m}}(-1+i)\xi_r\sqrt{dt}$$

$$rd\theta = -\frac{i\hbar}{m}\left(\frac{\partial \ln \Psi}{\partial \theta} + \frac{\cot \theta}{2r}\right)dt + \sqrt{\frac{\hbar}{2m}}(-1+i)\xi_\theta\sqrt{dt}$$

$$r\sin\theta d\phi = -\frac{i\hbar}{mr\sin\theta}\frac{\partial \ln \Psi}{\partial \phi}dt + \sqrt{\frac{\hbar}{2m}}(-1+i)\xi_\phi\sqrt{dt}$$

where  $(\xi_r, \xi_\theta, \xi_\phi)$  are three independent random variables with normal distribution. This stochastic model can be applied to the quantum system whose time evolution is described by  $\Psi(r, \theta, \phi)$ .