

From: Pedro Pessoa & Bruno Arderucio Costa
Re: Response to Reviewer # 1

We can see that the anonymous reviewer gave comprehensive comments to our paper. We are thankful for the time taken to check and fully understand each of our arguments and how they fit as replies to Tsallis' article and gave interesting suggestions on how to improve. Since they decided not to sign the review, we added a acknowledgment to an anonymous reviewer, but, if they request, we can change it to their name.

We are also thankful for the reviewer's comments on our writing, although we are both based on English-speaking countries neither of us is a native speaker. Because of this, their suggestions and corrections as specific comments were extremely valuable.

A summary of how the reviewer's suggestions were adopted can be seen below.

I. On the general comments

In view of these comments, this reader would very much appreciate if the Authors could better explain in their work why additivity cannot be sacrificed while extensivity is not an essential feature of thermodynamical entropy. Perhaps, it would also be a good idea to add an appendix where some sort of derivation of the BH entropy and its proportionality to the its area is sketched. Indeed, both Pessoa-Costa and Tsallis seem to agree that the use of the BG entropy for the computation of that area law is not obvious at all.

We completely agree to their first suggestion. We argued that KL/BG entropy should not be replaced for two reasons: (i) subsystem independence implies KL entropy, therefore maximizing a non-additive functional would lead to a statistics for which subsystems are going to be correlated even in the total absence of information (data or constraints) that indicate so; and (ii) The thermodynamical entropy derived from KL/BG entropy [E.T Jaynes *Am. J. Phys.* **1965**, 33, 391] physical description (Hamiltonian dynamics) of systems, and that is independently of extensivity. In order to improve the quality of the explanation in the main text the following changes were made:

- We added the laws of thermodynamics arguments from the highlighted text starting with "This result relies only on the fact that $S(\mathbf{A})$ is constructed" (line 47) – (previously this information was in footnote number [10]).
- We explicated our arguments (i) and (ii) with the highlighted text starting with "As explained, assigning probabilities using a functional that differs" (line 84).

On the second suggestion, we share the feeling that a sketch of the laws of Black Hole thermodynamics would improve the paper. However, due to the highly-technical nature of their derivations, we see that developing upon it will take a long explanation that escapes the scope of the present manuscript and will not add to our main arguments. Instead, we added a short paragraph starting in "It is important to point out that the laws of black hole mechanics" (line 118) specifying the hypotheses needed for the first law to hold, and an extra sentence starting in "And the identification of α " (line 126) making the connection with formula (5) more explicit.

The reviewer also pointed out

the nonextensive character of the (Boltzmann) entropy also has important consequences, including the loss of extensive character of energy and volume

but we honestly believe that this was already properly answered in the original manuscript. For a black hole, as a system with long-range interactions, the loss of extensivity in energy is not only not an issue, but expected. As we argued in the paragraph starting with "Second, a system bound by self-interaction " (line 75).

II. On the specific comments

- 1 Suggestion adopted.
- 2 Tr , $\hat{\rho}$, and $\hat{\phi}$ were appropriately specified.
- 3 We presented right after eq 4 (ln 39-41) that i are a indicator of the k sufficient statistics. And explained the distinctions between upper and lower indexes.
- 4 Correction adopted.
- 5 References added.
- 6 Correction adopted.
- 7 The text was modified so to refer to "natural units" for which the definition is given in footnote [21] (ln 100)
- 8 Reference added.
- 9 Correction adopted.
- 10 We replaced "We were able to invalidate" by "We contested"
- 11 We replaced "we show derive" by "we derive"
- 12 Bound specified.
- 13 Suggestion adopted.
- 14 Correction adopted.